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On Some Algebraic Properties of the Root Set of Multisets and Soft Multisets

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Abstract

In this paper, some inconsistencies in the use of operation symbols in msets, different from the classical set is first addressed. The paper presents a very important aspect of msets and soft multisets, which is their root set. It is established that the root set of the union of two msets is equal to the union of the two root msets, that the root set of the intersection of two msets is the same as the intersection of the root sets of the two msets, that the nth power of the union of two msets is equal to the union of the nth power of the individual msets, that the nth power of the intersection of two msets is the same as the intersection of the nth power of the two msets and several other results. Msets and soft msets are useful in many areas of mathematics, computer science, in decision making and such other areas that deals with uncertainties.

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Introduction

The fundamental of Cantorian set theory is that no element is allowed to appear more than once in a set. In this case, any collection, involving repetition, like in $\{a, b, a, c, d, b\}$ is not a set, and can only be regarded as one only if it is adjusted to take the form: $\{a, b, c, d\}$ that is, by deleting the repeated elements.

Coincidentally, in other areas of science, and in real life, this 'unrepeated' aspect of Cantorian set theory is not feasible especially when seeking mathematical representation of some concepts. In real life, instances like repeated roots in a polynomial equation, repeated observations in statistical repeated hydrogen atoms in a water molecule, $H_2O = \{H, H, O\} = \{2/H, 1/O\}$, the prime factorization of an integer $n > 0$, Zeros and poles of meromorphic functions, invariants of matrices in a canonical form are msets, abound. Again, words in a language are msets on the set of alphabets $\sum$, in these sets, repetitions are inevitable. (Singh et al., (2008)

Thus, the word multiset, which is shortened to *mset,* abbreviates the term "multiple-membership set" was developed to address repetitions in a set. Since a set is a well-defined collection of distinct objects, weakening the condition of 'well – definedness' in a set gave birth to Fuzzy Sets and weakening of the condition of 'distinctness' brought about the notion of multisets. Therefore, multisets are sets in which the repetition of elements is significant. Msets just like fuzzy set is a generalization of classical sets.

In 1981, mset was adopted to replace terms like list, heap, bunch, bag, sample, weighted set, occurrence set and fireset, which were used previously in different context to describe the concept of mset and it was N.G Bruijn who first suggested it to Knuth in a private conversation (Knuth, 1981).

The mset theory which generalizes set theory as a special case was introduced by Cerf *et al.* (1971). Syropoulos (2011) defined various operations on multiset. Babitha and John (2013) presented the idea of soft multiset. Girish and Sunil (2012), introduced the concepts of relations, function, composition, and equivalence in msets context.

Soft multisets were introduced in Akhazaleh *et al.* (2011) using the idea of universes. Since then, so many scholars have contributed to the development of the theory using different approaches. (Babitha and John, 2013; Balami and Ibrahim ,2013; Mukherjee *et al.*,2014); and Tokat and Osmanoglu, 2011).

Isah (2018) expounded on some of the properties of soft mset, like relative null, relative seminull, absolute, relative absolute and relative semi-absolute soft msets. In this work, the context of mset and soft mset theory is reviewed. The root set of mset and soft mset and some of its algebraic properties are presented. The paper is organized as follows: In section two, some basics of msets and msets operations are presented, and the inconsistencies in the use of operation symbols are adressed. In section three, various operations on soft

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msets theory are studied. And four summarily, evaluates the applications of msets and soft msets theory.

Basics of Mset theory

Mset theory (Knuth, (1981)

A mset (mset, for short) is an unordered collection of objects in which, elements are allowed to repeat unlike in a classical set. In other words, an mset is a collection to which elements may belong more than once. That is, a mset is a mathematical entity that is like a set, but it is allowed to contain repeated elements.

In other words, a mset A drawn from the set X is represented by a count function $C_A(x)$ and is defined as $C_A: X \longrightarrow \mathbb{N}$, where $\mathbb{N}$ represents the set of natural numbers including zero.

Here $C_A(x)$ is the number of occurrences of the element x in the mset A. The presentation of the mset A drawn from X = $\{x_1, x_2, x_3, \ldots x_n\}$ will be as A= $\{ x_1/a_1, x_2/a_2, \ldots x_n/a_n \}$ where $a_i = C_A(x_i)$ for i = 1, 2, … n in the mset A.

From here, we let the set of all finite msets over the set X be represented by $\mathfrak{M}(X)$.

Mset operations

Union of msets ($\dot{\cup}$) (Singh *et al.*,2007)

Let A, B $\in \mathfrak{M}(X)$. $A \dot{\cup} B$ is the mset defined thus:

$C_{A \dot{\cup} B}(x) = \max\{C_A(x), C_B(x)\}$.

Here, an object *z* occurring *a* times in *A* and *b* times in *B*, occurs maximum (*a,* b) times in A$\dot{\cup}$ B, if such a maximum exists; otherwise, the minimum of (a, b) is taken, which always exists.

For example, if *A* = [2, 3, 4, 4], B = [1, 4, 3, 3] then $A \dot{\cup} B$ = [1, 2, 3, 3, 4, 4].

Note that for a finite mset X, the maximum multiplicity of the element of X always exists. However, for certain infinite sets, the maximum multiplicity of elements of X does not exist.

Intersection ($\dot{\cap}$) (Singh *et al.*,2007)

A $\dot{\cap}$ B is the mset defined by

$C_{A \dot{\cap} B}(x) =$ minimum $\{C_A(x), C_B(x)\}$,

which is the intersection of two numbers. That is, an object *x* occurring *a* times in *A* and *b* times in *B*, occurs minimum (*a, b*) times in $A \dot{\cap} B$, which always exists.

If A = [3, 3, 3, 4, 4]; B = [1, 4, 3, 3], then A $\dot{\cap}$ B = [3, 3, 4]

Note hence that for any mset X, we have $\dot{\cap} X \subseteq \dot{\cup} X$.

Direct sum (Singh *et al.*,2007)

Let A, B $\in \mathfrak{M}(X)$. The direct sum of two msets, A $\oplus$ B or A $\uplus$ B is the mset defined by $C_{A \oplus B}(x) = C_A(x) + C_B(x)$, for any $x \in$ X . That is, an object x occurring 'a' times in A and 'b' times in B, occurs a + b times in A$\uplus$ B.

For example, if A = $[1, 1,1,1,1, 2, 2, 2, 2, 4, 4]$, B = $[1,1, 2, 2, 3, 3, 3, 4, 4]$

then A$\uplus$ B = $[1, 1, 1,1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 4, 4, 4, 4]$

Note: $|A \uplus B| = | A \dot{\cup} B | + | A \dot{\cap} B |$.

Power set of mset (Blizard, 1989)

For any mset U, we denote the set of all msubsets of U by P(U), and we call it the powerset of U. For example:

Suppose that P $([x, y]_{3,1}) = \{\emptyset, \{x\}, [x]_2, [x]_3, \{y\}, \{x, y\}, [x, y]_2, [x, y]_{3,1}\}$

Informally, if X is a 'set' with n distinct elements, then P(X) contains exactly 2^n distinct elements. If X is a 'mset' with n elements (repetitions counted), then P(X) contains strictly less than 2^n elements.

Direct Product of msets

Let A, B $\in \mathfrak{M}(X)$. The direct product of two msets A and B, denoted by A $\otimes$ B, is the mset consisting of all ordered pairs (a, b) such that a $\in$ A and b $\in$ B. Clearly,

$C_{A \otimes B}((a, b)) = C_A(a)\, C_B(b)$ or $|A \otimes B| = |A| \otimes |B|$.

For example, if A = [a, a, c] and B = [b, b] are two msets, then A $\otimes$ B is given by

[a, a, c] $\otimes$ [b, b] = [(a, b), (a, b), (a, b), (a, b), (c, b), (c, b)] = $[(a, b), (c, b)]_{4,2}$

Arithmetic difference (Wildberger, 2003)

Let A, B $\in \mathfrak{M}(X)$, then $C_{A\ominus B}(x) = C_A(x) - C_{A\cap B}(x) \vee 0 \ \forall\, x \in X$. This is called the arithmetic difference of B from A. Note that even if B is not contained in A, this definition holds good.

If $B \subseteq A$, then $C_{A\ominus B}(x) = C_A(x) - C_B(x)$

Example 1; A = $[x, y]_{5,7}$ and B = $[x, y]_{3,5}$ then

$A - B = [x, y]_{2,2}$.

$Remark$: In the above example, A − B = $[x, y]_{2,2} \subseteq B$ contradicting the classical law:

(A − B) $\cap B = \emptyset$

mset symmetric difference (Petrovsky, (1997)

Let A, B $\in \mathfrak{M}(X)$. The symmetric difference, denoted $A\Delta B$, is defined by

$C_{A\triangle B}(x) = |\, C_A(x) - C_B(x) \,|$.

Note: $A\Delta B = (A - B) \cup (B - A)$.

mset complement (Petrovsky, (1997)

Let $G = \{A_1, A_2, \ldots, A_n\}$ be a family of finite msets generated from the set X. Then, the maximum mset Z is defined by $C_Z(x) = \ \bigvee_{A\in G} C_A(x)$ for all $A \in G$ and $x \in X$. The Complement of an mset A, denoted by $\bar{A}$, is defined:

$\bar{A} = Z - A \ such\ that \quad C_{\bar{A}}(x) = C_z(x) - C_A(x), for\ all\ x \in X.$

Note that $A_i \subseteq Z\ for\ all\ i.$

Multiplication by Scalar (Petrovsky, (1997)

Let $A \in \mathfrak{M}(X)$ and $k \in \{0,1,2, \ldots)$ be scalar, then the scalar multiplication denoted by k.A is defined by

$C_{kA}(x) = k.\, C_A(x)$

Remark: When $k = 0, C_{kA}(x) = 0, \forall\, x \in X$

$\Rightarrow kA = \emptyset$, *since* $C_{kA}(x) = 0 = C_{\emptyset}(x)$

$C_{kA}(x) = C_{\emptyset}(x) \Rightarrow kA = \emptyset$ (since k is a scalar, k$\neq \emptyset, \Rightarrow A = \emptyset$)

Raising to an arithmetic power: (Parker – Rhodes, (1981)

Let $A \in \mathfrak{M}(X)$. Then raising A to arithmetic power n denoted

A^n is given by

$C_{A^n}(x) = (C_A(x))^n$ for n $\in \{0, 1, 2, \ldots\}$

2.2.11 Arithmetic Multiplication (Petrovsky, (1997))

Let A, B $\in \mathfrak{M}(X)$, then the Arithmetic Multiplication denoted by $A.B$ is defined by

$C_{A.B}(x) = C_A(x).\, C_B(x) \ \forall\, x \in X.$

2.2.12 Equal msets: Let A, B $\in \mathfrak{M}(X)$. Two msets A and B are equal or the same, written as $A \doteq B, iff$ for any $x \in X, C_A(x) = C_B(x)$. That is, $A \doteq B$ if the multiplicity of every element in A is equal to its multiplicity in B and conversely.

Note: A $\doteq$ B $\Rightarrow A^* \doteq B^*$, however the converse need not hold, that is given the equality of root sets of A and B, the equality of the msets A and B is not automatic.

2.3 Some properties of mset operations (Knuth, 1981)

Let A, B $\in \mathfrak{M}(X)$, then the following properties hold:

i) Commutativity:

a) A⊎B $\doteq$ B⊎A

b) A$\dot{\cup}B \doteq$ B $\dot{\cup}$ A

c) A $\dot{\cap}$ B $\doteq$ B$\dot{\cap}$ A

ii) Associativity:

$a)\ A \uplus (B \uplus C) \doteq (A \uplus B) \uplus C$
$b)\ A \dot\cup (B \dot\cup C) \doteq (A \dot\cup B) \dot\cup C$
$C)\ A \dot\cap (B \dot\cap C) \doteq (A \dot\cap B) \dot\cap C$
iii) Idempotency
$A \dot\cup A \doteq A$; $A \dot\cap A \doteq A$; but $A \uplus A \not\doteq A$
iv) Identity law:
a) $A \dot\cup \emptyset \doteq A$; $A \dot\cap \emptyset \doteq \emptyset$; $A \uplus \emptyset \doteq A$,
v) Distributivity law:
a) $A \uplus (B \dot\cup C) \doteq (A \uplus B) \dot\cup (A \uplus C)$
b) $A \uplus (B \dot\cap C) \doteq (A \uplus B) \dot\cap (A \uplus B)$
c) $A \dot\cup (B \dot\cap C) \doteq (A \dot\cup B) \dot\cap (A \dot\cup B)$
d) $A \dot\cap (B \dot\cup C) \doteq (A \dot\cap B) \dot\cup (A \dot\cap C)$
It is easy to see that $\uplus$ is stronger than both $\dot\cup$ and $\dot\cap$ in the sense that neither distributes over $\uplus$, whereas $\uplus$ distributes over both $\dot\cup$ and $\dot\cap$.
So, $\dot\cap x \,\dot\subseteq\, \dot\cup x \,\dot\subseteq\, \uplus x$
Root or support mset (Cerf et. al., (1971))
Let $M \in \mathfrak{M}(X)$. The support set of M denoted by M^* is a subset of X given by
$M^* = \{x \in X: C_M(x) > 0\}$. M^* is also called root set.

Proposition 1
Let M, N $\in \mathfrak{M}(X)$ such that M^* and N^* are the root sets. Then the following holds:
$(M \dot\cup N)^* \doteq M^* \dot\cup N^*$
$(M \dot\cap N)^* \doteq M^* \dot\cap N^*$
$(M \dot\cup N)^n \doteq M^n \dot\cup N^n$
$(M \dot\cap N)^n \doteq M^n \dot\cap N^n$ $(\forall\, n = 0,1,2,3, \ldots)$
Proof
Let $x \in (M \dot\cup N)^*$
then $C_{M \dot\cup N}(x) > 0$
$i.e$ Max $\{C_M(x), C_N(x)\} > 0$
$i.e$ either $C_M(x) > 0$ or $C_N(x) > 0$
$then$ either $x \in M^*$ or $x \in N^*$
$thus$ $x \in M^* \dot\cup N^*$
$thus$ $(M \dot\cup N)^* \subseteq M^* \dot\cup N^*$ ______________________________ (1)
Now let y $\in M^* \dot\cup N^*$
we have $y \in M^*$ or $y \in N^*$
$i.e$ $C_M(y) > 0$ or $C_N(y) > 0$
thus Max $\{C_M(y), C_N(y)\} > 0$
in particular $C_{M \dot\cup N}(y) > 0$
$hence$ $y \in (M \dot\cup N)^*$
and $M^* \dot\cup N^* \subseteq (M \dot\cup N)^*$ ___________________________ (ii)
$from$ (i) and (ii) we have
$M^* \dot\cup N^* \doteq (M \dot\cup N)^*$
2. Let $x \in (M \dot\cap N)^*$
$then$ $C_{M \dot\cap N}(x) > 0$
$i.e$ min $\{C_M(x), C_N(x)\} > 0$
then $C_M(x) > 0$ and $C_N(x) > 0$

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then $x \in M^*$ and $x \in N^*$
thus $x \in M^* \dot{\cap} N^*$
$i.e$ $(M\dot{\cap}N)^* \dot{\subseteq} M^* \dot{\cap} N^*$ ______________________________ (1)
similarly Let $y \in M^* \dot{\cap} N^*$
thar is $y \in M^*$ and $y \in N^*$
In particular, $C_M(y) > 0$ and $C_N(y) > 0$
min $\{C_M(y) > 0, C_N(y)\} > 0$
$i.e$, $C_{M\dot{\cap}N}(y) > 0$
That is, $y \in (M\dot{\cap}N)^*$
Thus $M^* \dot{\cap} N^* \dot{\subseteq} (M\dot{\cap}N)^*$ ______________________________ (2)
Comparing (i) *and* (ii) we have $(M\dot{\cap}N)^* \doteq M^* \dot{\cap} N^*$
3. Now for any $x \in X$
We have $C_{(M\dot{\cup}N)^n}(x) = (C_{(M\dot{\cup}N)}(x))^n$ (by definition)
But $C_{(M\dot{\cup}N)}(x) = \max\{C_M(x), C_N(x)\}$
Then $(C_{(M\dot{\cup}N)}(x))^n = (\max\{C_M(x), C_N(x))^n$
$= \max\{(C_M(x))^n, (C_N(x))^n\}$
$= \max\{C_{M^n}(x), C_{N^n}(x)$
$= C_{M^n\dot{\cup}N^n}(x)$
Thus $(M\dot{\cup}N)^n \doteq M^n\dot{\cup}N^n$
4. For any $x \in X$
$C_{(M\dot{\cap}N)^n}(x) = (C_{M\dot{\cap}N}(x))^n$ (by definition)
but $C_{M\dot{\cap}N}(x) = \min\{C_M(x), C_N(x)\}$
thus $(C_{M\dot{\cap}N}(x))^n = (\min\{C_M(x), C_N(x)\})^n$
$= \min\{(C_M(x))^n, (C_N(x))^n\}$
$= \min\{C_{M^n}(x), C_{N^n}(x)\}$
$= C_{M^n\dot{\cap}N^n}(x),$
Then $(M \dot{\cap} N)^n \doteq M^n \dot{\cap} N^n$

Soft mset

Soft Mset (Soft Mset) (Osmanoglu and Tokat, 2014; Tokat *et al.*, 2015)

Let X be a universal set, M be an mset over X, E be a set of parameters and A $\dot{\subseteq}$ E. Then a pair (F, A) is called a soft mset over M where $F: A \longrightarrow P^*(X)$. For all $\alpha \in A$, the mset $F(\alpha)$ is represented by a count function $C_{F(\alpha)}: X^* \longrightarrow \mathbb{N}$.

The set of all submsets of M, denoted by $P^*(X)$ is called the power mset of M.

Let X be a universal set, and $\mathfrak{M}$ the set of all finite mset over the set U. We denote the set of all finite soft msets over U by $S\mathfrak{M}(X)$.

Soft mset operations

Let $(F, A), (G, B) \in S\mathfrak{M}(X)$. Then the following holds:

Soft submset

$(F$, A) is a soft sub mset of $(G$, B) written as $(F$, A) $\dot{\subseteq} (G, B)$ if the following conditions are satisfied:

(a) $A \subseteq B$

(b) $C_{F(\alpha)}(x) \leq C_{G(\alpha)}(x), \forall x \in U^*, \forall \alpha \in A$.

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Soft equal mset

$(F, A) \doteq (G, B)$ iff $(F, A) \dot{\subseteq} (G, B)$ and $(G, B) \dot{\subseteq} (F, A)$.

Also, if $(F, A) \dot{\subset} (G, B)$ and $(F, A) \dot{\neq} (G, B)$ then (F, A) is called a proper soft submset of (G, B).

Soft Union:

$(F, A) \dot{\cup} (G, B) \doteq (H, C)$ where $C = A \dot{\cup} B$ and

$C_{H(\alpha)}(x) = \max\{C_{F(\alpha)}(x), C_{G(\alpha)}(x)\}, \forall \alpha \in C, \forall x \in U^*$.

Soft Intersection:

$(F, A) \dot{\cap} (G, B) \doteq (H, C)$ where $C = A \dot{\cap} B$ and

$C_{H(\alpha)}(x) = min\{C_{F(\alpha)}(x), C_{G(\alpha)}(x)\}, \forall \alpha \in C, \forall x \in U^*$.

Soft Difference:

$(F, A) \backslash (G, B) \doteq (H, C)$ where $C_{H(\alpha)}(x) = max\{C_{F(\alpha)}(x) - C_{G(\alpha)}(x), 0\}, \forall x \in U^*$.

Soft Null:

A soft mset (F, A) is called a Null soft mset denoted by $\emptyset$ if $\forall \alpha \in A$, F$(e)=\emptyset$.

Soft Complement:

The complement of a soft mset (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \longrightarrow P^*(U)$ is a mapping given by $F^c(\alpha) = U \backslash F(\alpha), \forall \alpha \in A$ where $C_{F^c(\alpha)}(x) = C_U(x) - C_{F(\alpha)}(x), \forall x \in U^*$

Root set of a soft mset

Let (F, A) $\in S\mathfrak{M}(X)$.

The root set of (F, A) denoted by $(F, A)^*$ is given by $(F, A)^* = (F^*, A)$ where $F^*: A \rightarrow P(U^*)$ given by $F^*(\alpha) = (F(\alpha))^*$

Proposition 2: Let (F, A) $(G, B) \in S\mathfrak{M}(X)$. Then

$((F, A) \dot{\cup} (G, B))^* \doteq (F, A)^* \dot{\cup} (G, B)^*$

$((F, A) \dot{\cap} (G, B))^* \doteq (F, A)^* \dot{\cap} (G, B)^*$

Proof:

Let (F, A), $(G, B) \in S\mathfrak{M}(X)$.

Let $x \in ((F, A) \dot{\cup} (G, B))^*$ for any $\alpha \in A \tilde{\cup} B$

$then\ C_{(F \dot{\cup} G)^*(\alpha)}(x)$

$i.e\ C_{((F)^*(\alpha) \dot{\cup} G^*(\alpha))}(x)$

$\max\{C_{F^*(\alpha)}(x), C_{G^*(\alpha)}(x)\}$

$i.e\ x \in (F, A)^*$ or $x \in (G, B)^*$

that is either $x \in (F, A)^*$ or $x \in (G, B)^*$

$that\ is\ x \in (F, A)^* \dot{\cup} (G, B)^*$

Thus, $((F, A) \dot{\cup} (G, B))^* \subseteq (F, A)^* \dot{\cup} (G, B)^*$............................. (1)

Now let $y \in (F, A)^* \dot{\cup} (G, B)^*$ for any $\alpha \in A \dot{\cup} B$

$C_{((F)^*(\alpha) \dot{\cup} G^*(\alpha))}(x)$

$= C_{(F(\alpha))^* \dot{\cup} (G(\alpha))^*}(x)$

$= C_{(F(\alpha) \dot{\cup} G(\alpha))^*}(x)$

$i.e\ y \in ((F, A) \dot{\cup} (G, B))^*$

$(F, A)^* \dot{\cup} (G, B)^* \subseteq ((F, A) \dot{\cup} (G, B))^*$............................ ......(2)

By (1) and (2)

$((F, A) \dot{\cup} (G, B))^* = (F, A)^* \dot{\cup} (G, B)^*$

Let (F, A) $(\mathrm{G}, \mathrm{B}) \in S\mathfrak{M}(X)$.
Let $x \in$ ((F, A) $\dot{\cap}$(G, B))* for any $\alpha \in A \dot{\cap} B$
$then\ C_{(F \dot{\cap} G)^*(\alpha)}(x)$
$i.e\ C_{((F)^*(\alpha) \dot{\cap} G^*(\alpha))}(x)$
$= \min\{C_{F^*(\alpha)}(x), C_{G^*(\alpha)}(x)\}$
$i.e\ x \in (F, A)^*$ and $x \in (G, B)^*$
that is either $x \in (F, A)^*$ and $x \in (G, B)^*$
$that\ is\ x \in (F, A)^* \dot{\cap} (G, B)^*$
((F, A) $\dot{\cap}$ (G, B))$^* \subseteq (F, A)^* \dot{\cap}$ (G, B)*............................ (1)
Now let y $\in (F, A)^* \dot{\cap}$ (G, B)* for any $\alpha \in A \dot{\cap} B$
That is, $C_{((F)^*(\alpha) \dot{\cap} G^*(\alpha))}(x)$
$= C_{(F(\alpha))^* \dot{\cap} (G(\alpha))^*}(x)$
$= C_{(F(\alpha) \dot{\cap} G(\alpha))^*}(x)$
$i.e\ y \in (F, A)^*$ and $y \in (G, B)^*$
$\Longrightarrow y \in (F(\alpha))^* \dot{\cap} (G(\alpha))^*$
$thus\ y \in ((F, A) \dot{\cap} (G, B))^*$
$(F, A)^* \dot{\cap}$ (G, B)$^* \subseteq$ ((F, A) $\dot{\cap}$ (G, B))*............................ (2)
From (1) and (2)
((F, A) $\dot{\cap}$ (G, B))$^* \doteq (F, A)^* \dot{\cap}$ (G, B)*

Some properties of soft multisets (Isah, 2018)
Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then
(i) (F, A) is called a relative null soft multiset with respect to A denoted by
$(F, A)_\emptyset$ if $F(e) = \emptyset, \forall e \in A$.
(ii) (F, A) is called a relative semi-null soft multiset with respect to A, denoted by
$(F, A)_{\emptyset 1}$ if $\exists\ e \in A$ such that F(e)=$\emptyset$.
(F, A) is called a relative absolute soft multiset with respect to A denoted by
$(F, A)_U$ if F(e)=U,$\forall e \in A$.
(F, A) is called a relative semi-absolute soft multiset with respect to A, denoted bu
$(F,A)_{\mathrm{U}1}$ if $\exists\ e \in A$ such that F(e)=U.
(v) (F, A) is said to be the absolute soft multiset denoted $(F, A)_{\mathrm{U}}$ if $F(e)$=U,$\forall e \in E$.

Example (Isah, 2018)
Let $U = \{5/w, 2/x, 3/y\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and $A = \{e_1, e_2, e_7\}$.
Then $(i)\ If\ F(e_1) = \emptyset$,
$F(e_2) = \emptyset, F(e_7) = \emptyset$, then $(F, A)$is a relative null soft mset with respect to A
$(ii)\ If\ F(e_1) = \{2/w, 1/y\}$, $F(e_2) = \emptyset$
$F(e_7) = \{4/w, 1/y, 1/x\}\ then\ (F, A)$ is a relative semi-null soft set $with\ respect\ to\ A.$
$(iii) If\ F(e_1) = \{5/w, 2/x, 3/y\}, F(e_2) = \{5/w, 2/x, 3/y\}$
$F(e_7) = \{5/w, 2/x, 3/y\}, then\ (F, A)$ is a relative absolute soft multiset.
$(iv)\ If\ F(e_1) = \{5/w, 2/x, 3/y\}, F(e_2) = \{1/x, 3/y\}$
$F(e_7) = \{4/w, 2/x, 1/y\}, then\ (F, A)$ is a relative semi-absolute soft mset with respect to A

$$(v) If\ F(e_1) = \left\{\frac{5}{w}, \frac{2}{x}, \frac{3}{y}\right\}, F(e_2) = \left\{\frac{5}{w}, \frac{2}{x}, \frac{3}{y}\right\},$$

$$F(e_3) = \left\{\frac{5}{w}, \frac{2}{x}, \frac{3}{y}\right\},$$
$$F(e_4) = \left\{\frac{5}{w}, \frac{2}{x}, \frac{3}{y}\right\},$$
$$F(e_5) = \left\{\frac{5}{w}, \frac{2}{x}, \frac{3}{y}\right\},$$
$$F(e_6) = \left\{\frac{5}{w}, \frac{2}{x}, \frac{3}{y}\right\},$$

$F(e_7)=\left\{\frac{5}{w}, \frac{2}{x}, \frac{3}{y}\right\}$

Then (F, E) is the absolute soft multiset with respect to E.

Some applications of soft mset

The theory of mset has applications in many fields such as mathematics, computer science and social sciences amongst others (Blizard, 1991; Singh *et al.*, 2007; Isah and Tella, 2015; Singh and Isah, 2015). Msets are useful in Mathematics and Computer Science as seen in prime number theorem, the prime factors and others. So, to represent the prime factors of a given number, this can only be represented by a mset but not a set.

Msets have been applied in statistics, multicriteria decision making, knowledge representation in data-based systems, biological systems and membrane computing (Yager, 1987; Miyamoto 2003; Paun and Perez-Jimenez, 2006; Kosters and Laros, 2007).

Msets are used for proving termination properties and in some search and sort algorithms (Blizard, 1991), other notable uses of msets are seen in the theory of Petri nets by Peterson (Peterson, 1981), relational databases by Yager (Yager, 1987), automata theory by Eilenberg and software verification in (Eilenberg, 1974).

Conclusion

The paper presents in part, a very important aspect of mset and soft multisets, which is their root set. It is established that the root set of the union of two msets is equal to the union of the two root msets and several other results. Msets and soft msets are useful in many areas of mathematics, computer science, decision making and such other areas that deals with uncertainties.

References

Alkhazaleh, S., Salleh, A. R. and Hassan, N. (2011). Soft Multisets Theory, Applied Mathematical Sciences, 5 (72) 3561-3573.

Babitha, K. V. and John, S.J. (2013). On soft multi-set, Annals of Fuzzy Mathematics and Informatics, 5(1), 35-44.

Balami ,H.M. and Ibrahim, A.M. (2013). Soft multiset and its application in information system, International Journal of scientific research and management, 1(9), 471-481.

Blizard, W. (1991). The Development of Mset Theory, *Modern Logic*, 1: 319-352.

Blizard, W. (1989). Mset Theory, Notre Dame Journal of Formal Logic, 30, 36-66.

Cerf, V., Fenandez, E., Gostelow, K. and Volausky, S.(1971). Formal control and low properties of a model of computation, Report ENG 7178, Computer Science Department, University of California, Los Angeles, CA, pp. 81,

Girish, K. P. and John, S. J. (2012). Mset Topologies Induced by Mset Relations, *Information Science,* 188: 298-313.

Isah, A. I. (2018). Some Properties of Soft Mset *Science World Journal* Vol 13(No 4)

Isah, A. I. and Tella, Y. (2015). The Concept of Mset Category, *British Journal of Mathematics & Computer Science* 9 (5) 427-437.

Kosters, W.A and Laros J. F. J. (2007). Metrics for mining msets, in: 27th SGAI International Conference on Innovative Techniques and Applications of Artificial Intelligence, 2007, pp.293-303.

Knuth, D.E. (1981). The art of computer programming, vol 2: Semimimerical Algorithms. Second Edition, Addison-Wesley.

March, Volume 10, Number 1, Pages 24 – 32 http://www.ijbst.fuotuoke.edu.ng/ 32
https://doi.org/ **10.5555/QWSD5182** ISSN 2488-8648

Miyamoto, S. (2003). Two generalizations of msets, in: M. Inuiguchi, S. Tsumito eds., Rough Set theory and Granular computing, Springer, pp 59-68.

Mukherjee, A., Das, A. K. and Saha, A. (2014). Topological structure formed by soft multi sets and soft multi compact space, *Annals of Fuzzy Mathematics and Informatics*, 7(6), 919-933.

Osmanoglu, I. and Tokat, D. (2014). Compact Soft Multi Spaces, *European Journal of Pure and Applied Mathematics*, 7 (1) 97-108.

Paun G. and Perez-Jimenez M.J. (2006), Membrane computing: brief introduction, recent results and applications, *Bio Systems* 85; 11-22.

Petrovsky, A. B. (1997). Structuring Techniques in Multiset Spaces. Springer. 174-184.

Singh, D. and Isah, A. I. (2015). Some Algebraic Structures of Languages, *Journal of mathematics and computer science* 14: 250-257.

Singh, D., Ibrahim, A. M., Yohanna, T. and Singh, J. N. (2008). A systematization of fundamentals of msets 29 : pp 33- 48.

Syropoulos, A. (2011) . Mathematics of Multisets, GR-671 00 Xanith, GREECE.

Tokat ,D. and Osmanoglu, I. (2011). Soft multi set and soft multi topology, Nevsehir Universitesi Fen Bilimleri Enstitusu Dergisi Cilt, 2, 109-118.

Tokat, D., Osmanoglu, I. and Ücok, Y. (2015). On Soft Multi Semi-continuous Functions, *Annals of Fuzzy Mathematics and Informatics*, 10 (5) 691-704.

Wildberger, N. J. (2003). A new look at mset, School of Mathematics, UNSW Sydney 2052,Australia.

Yager, R.R (1986). On the theory of bags, *Internat. J. Gen. Systems* 13(1) 23-37.