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Improved Exponential Ratio-In-Regression Class of Estimators of the Population Mean in Simple Random Sampling in the presence of Two Auxiliary Variables

Ekpenyong, E. J. and Enang, E.I.
Department of Statistics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria

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Abstract

This study proposes two classes of exponential ratio estimators of the population mean under simple random sampling, in the presence of two auxiliary variables. Their biases and mean square errors have been derived using first and second degree approximation. It is observed that some existing estimators are members of this class of estimators of population mean. Theoretical and empirical comparisons of the proposed multivariate estimators with existing ones, at optimal conditions show greater gains in efficiencies over existing multivariate estimators of the population mean considered in this work.

*Corresponding Author: Ekpenyong, E. J.; **ekpesstat@yahoo.com**

INTRODUCTION

The study of ratio and regression estimators has also been extended to the case where multi-supplementary information are employed for further improvement of precision of estimators of population parameters. Notable contributions in this direction include Swain (2012), Lu and Yan (2011), Kadilar and Cingi (2004b, 2005), Abu-Dayyeh, Ahmed *et al.* (2003), Singh and Chaudhury (1986), and Khare *et al.* (2013) for simple random sampling scheme.

Many authors have used two or more auxiliary variables to increase the precision of estimation with most of the works limited to two auxiliary variables. Notable in the list is the work of Olkin (1958), which was reflected in Singh and Chaudhary (1986). Singh (1965, 1967) proposed ratio-cum-product estimators of population mean using two auxiliary variables. These estimators had higher efficiencies than the classical estimators of population mean using a single auxiliary variable. Perri (2004) was able to show that the Singh estimators are however less efficient than the traditional regression estimator of population mean using two auxiliary variables. Perri (2005) further modified the estimators of Singh (1965, 1967) to have estimators, whose Mean Square Errors were similar to the traditional multivariate regression estimator of population mean and hence more efficient than Singh (1965, 1967) estimators. Kadilar (2004, 2005) suggested new multivariate ratio estimators with improved efficiencies in some populations. They went further to establish conditions under which their estimators would be better than the classical estimators. Abu-Dayyeh *et al.* (2003) proposed two multivariate ratio estimators of population mean, but one of them showed higher efficiency than the other. Diana and Perri (2007) reviewed some contributions of authors to the improvement of efficiency of multivariate ratio and regression estimators. They concluded by proposing a class of multivariate estimators that made use of variances of the auxiliary variables instead of their means. They finally deduced that these set of estimators showed tremendous gains in efficiencies.

Further attempts to still improve on the estimation procedures of population mean using two auxiliary variables include works of Tailor, Verma and Sharma (2010), Riaz, Noor-ul-Amin and Hanif (2014), Lu,Yan and Peng (2014), Lu (2013), Singh and Singh (2014), Lu and Yan (2014), Rashid, Noor-ul-Amin, and Hanif (2015) and Swain (2012). Some of their estimators such as Riaz *et al.*(2014), Lu and Yan (2014), Rashid *et al.* (2014), Swain (2012) have the same efficiencies as Abu-Dayyeh *et al.*(2003) and Kadilar and Cingi (2005). A significant contribution also came from the works of Yasmeen, Noor-ul-Amin and Hanif (2015) who proposed a multivariate estimator through a linear combination of ratio-cum-exponential ratio and product-cum-exponential product estimators of population mean with two auxiliary variables. Although their estimator showed a significant gain in efficiency of the estimator in some populations over some of estimators mentioned earlier, some of the parameters were not properly

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determined. On this note, there is still the need for a greater improvement on the efficiency of multivariate estimators. This work is an attempts in this direction..

Review of some related existing estimators

Consider a finite population $U = (U_{1,} U_2, \ldots, U_N)$ of size(N). Let $(X_1), (X_2)$ and (Υ) denote the auxiliary and study variables taking values X_{1i}, X_{2i} and Υ_i respectively on the i^{th} unit $U_i (i = 1, 2, \ldots, N)$ population. It is assumed that $(x_{1i}, x_{2i}, y_i) \geq 0$, (since survey variables are generally non-negative) .

and information on the population means $(\bar{X}_1, \bar{X}_2)$ of the auxiliary variables is known. Let a sample of size (n) be drawn by simple random sampling without replacement (SRSWOR) from which we obtain the means $(\bar{x}_1), (\bar{x}_2)$ and $(\bar{y})$ for the auxiliary variables (X_1, X_2) and the study variable(Υ). For the above population we provide below a summary of some existing estimators with its mean square error. Some of the existing and related estimators under this form are presented in Table 1

Table 1: Some Existing Multivariate estimators of population mean in Simple Random Sampling and their MSEs

S/N	Estimator	MSE
1	$\theta_1^* \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right) + \theta_2^* \bar{y}\left(\frac{\bar{X}_2}{\bar{x}_2}\right)$ Traditional multivariance Ratio Singh and Chaudhary (1986)	$\lambda \bar{Y}^2\{C_y^2 + \theta_1^{*2} C_{x1}^2 + \theta_2^{*2} C_{x2}^2 - 2\theta_1^* \rho_{yx1} C_y C_{x1} - 2\theta_1^* \rho_{yx2} C_y C_{x2} + 2\theta_1^* \theta_2^* \rho_{x_1x_2} C_{x_1} C_{x_2}\}$
2	$\bar{y} + b_{yx1}(\bar{X}_1 - \bar{x}_1) + b_{yx2}(\bar{X}_2 - \bar{x}_2)$ Regression Estimator	$\lambda \bar{Y}^2 C_y^2 (1 - \rho_{yx1}^2 - \rho_{yx2}^2 + 2\rho_{yx1}\rho_{yx2}\rho_{x_1x_2})$
3	$\bar{y}_a = \bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{a_1} \left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{a_2}$ $\bar{y}_\epsilon = \epsilon_1 \bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\propto_1} + \epsilon_2 \bar{y}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\propto_2}$ Abu – Dayyeh *et al*. (2003)	$\lambda \bar{Y}^2 \left\{1 - \frac{\rho_{yx1}^2 + \rho_{yx2}^2 - 2\rho_{yx1}\rho_{yx2}\rho_{x_1x_2}}{1 - \rho_{x_1x_2}^2}\right\}$ $\lambda \bar{Y}^2 \{C_y^2 + \epsilon_1^{*2} (a_1^2 C_{x1}^2 - a_2^2 C_{x2}^2 - 2a_1 a_2 \rho_{x_1x_2} C_{x1} C_{x2}) + a_2^2 C_{x2}^2 +$ $-2 \epsilon_1^* (a_2^2 C_{x2}^2 - a_1 \rho_{yx1} C_y C_{x1} + a_2 \rho_{yx_2} C_y C_{x2} - a_1 a_2 \rho_x$
4	$\bar{y}_{MKC} = \bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\alpha_1} \left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\alpha_2} + b_{yx1}(\bar{X}_1 - \bar{x}_1) + b_{yx2}(\bar{X}_2 - \bar{x}_2)$ Kadilar and Cingi (2005)	$\lambda C_y^2 \bar{Y}^2\{1 + C_1^2 + C_2^2 + 2C_1 C_2 \rho x_1 x_2 - 2C_1 \rho_y x_1 - 2C_2 \rho_y x_2\}$
5	$\bar{y}_{LY} = W_{1l} \bar{y} \gamma_i + W_{2l} \bar{y} \gamma_j$ $a_{1l} = 0.43, b_{1l} = 0.78, a_{2l} = 1.22, b_{2l} = 0.72$ $\bar{y}_S = \left[\bar{y} + \emptyset_{opt}(\bar{X}_1 - \bar{x}_1)\left(\frac{\bar{X}_2}{\bar{x}_2}\right)^{\propto'}\right]$ Lu and Yan (2014)	$\lambda \bar{Y}^2\{C_y^2 + W_{1l}^{*2} \propto_{1l}^2 C_{x1}^2 + W_{2l}^{*2} \propto_{2l}^2 C_{x2}^2 - 2W_{1l}^* \propto_{1l} \rho_{yx_1} C_y C_{x1} - 2W_{2l}^* \propto_{2l} \rho_{yx_2} C_y C_{x2} + 2W_{1l}^* W_{2l}^* \propto_{1l} \propto_{2l} \rho_{x_1x_2} C_{x1} C_{x2}\}$ $\lambda \bar{Y}^2 C_y^2 (1 - R_{y.x_1x_2}^2)$
6	$\bar{y}_{alcr} = \bar{y}\left(\frac{W_{l1}\bar{X}_1 + W_{l2}\bar{X}_2}{W_{l1}\bar{x}_1 + W_{l2}\bar{x}_2}\right)^{\propto_l}$ $\bar{y}_{lcreg} = \bar{y} + b_{lC}(\bar{X}_{lC} - \bar{x}_{lC})$ Lu (2013)	$\lambda \{\bar{Y}^2 C_y^2 + 4W_{1l}^{*2} \propto_l^2 R_{lC}^2 C_{x1}^2 \bar{X}_1^2 + 4W_{2l}^{*2} \propto_l^2 R_{lC}^2 C_{x2}^2 \bar{X}_2^2 - 2W_{1l}^{*2} \propto_l \rho_{yx1} C_y C_{x1} - 2W_{2l}^* \propto_l R_{lC} \rho_{yx2} C_y C_{x2} + 4W_{1l}^* W_{2l}^* \propto_l^2 R_{lC}^2 \rho_{x1x2} C_{x1} C_{x2}\}$ $\lambda \bar{Y}^2 C_y^2 (1 - \rho_{yxlc}^2)$
7	$\bar{y}_{lcr} = \bar{y}\, exp\left[\frac{\bar{X}_{lcr} - \bar{x}_{lcr}}{\bar{X}_{lcr} + \bar{x}_{lcr}}\right]$ Lu *et al* . (2014)	$\lambda \{\bar{Y}^2 C_y^2 + \frac{1}{4} W_{1lcr}^{*2} R_{lcr}^2 \bar{X}_1^2 C_{x_1}^2 + \frac{1}{4} W_{2lcr}^{*2} R_{lcr}^2 \bar{X}_2^2 C_{x_2}^2 - \frac{1}{2} W_{1lcr}^* R_{lcr}^* \rho_{yx_1} C_y C_{x_1} \bar{X}_1 \bar{Y}_1 - \frac{1}{2} W_{2lcr}^* R_{lcr}^* \rho_{yx_2} C_y C_{x2} \bar{X}_2 \bar{Y} + \frac{1}{4} W_{1lcr}^* W_{2lcr}^* R_{1cr}^* \rho_{x_1x_2} C_{x_1} C_{x_2} \bar{X}_1 \bar{X}_2\}$
8	$\bar{y}_{RZ} = [\bar{y} + b_{yx1}(\bar{X}_2 - \bar{x}_2)] exp\left[\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 - \bar{x}_1}\right]$	$\frac{\lambda \bar{Y} C_y^2}{1 - \rho_{x1x2}^2}[1 - \rho_{yx1}^2 - \rho_{yx2}^2 + 2\rho_{yx1}\rho_{yx2}\rho_{x1x2}]$

9	Riaz *et al* .(2014) $\bar{y}_y = w_y\bar{y} + (1-w_y)\bar{y}\left(\frac{\bar{x}_{1t}}{\bar{X}}\right)\exp\left[\alpha_y\frac{\bar{x}_{2t}-\bar{X}_2}{\bar{x}_{2t}+\bar{X}_2}\right]$ Yasmeen *et al*. (2015)	$\lambda\bar{Y}^2\left\{C_y^2 + \frac{\propto_y^2}{4}g^2C_{x_2}^2(w_y-1)^2 + (w_y-1)^2g^2C_{x_1}^2 + (w_y-1)\propto_y gC_yC_{x_2}\rho_{yx2} + (w_y-1)^2g^2C_{x_1}C_{x_2}\rho_{x_1x_2} + 2(w_y-1)\propto_y gC_yC_{x_1}\rho_{yx_1}\right\}$

where

$$\bar{X}_1 = N^{-1}\sum_{i=1}^{N}x_{1i}\text{ , population mean of the first auxiliary variable}$$

$$\bar{X}_2 = N^{-1}\sum_{i=1}^{N}x_{2i}\text{ , population mean of the second auxiliary variable}$$

$$\bar{Y} = N^{-1}\sum_{i=1}^{N}y_i\text{ , population mean of the study variable}$$

$$\bar{x}_1 = n^{-1}\sum_{i=1}^{n}x_{1i}\text{ , sample mean of the first auxiliary variable}$$

$$\bar{x}_2 = n^{-1}\sum_{i=1}^{n}x_{2i}\text{ , sample mean of the second auxiliary variable}$$

$$\bar{y} = n^{-1}\sum_{i=1}^{n}y_i\text{ , sample mean of the study variable}$$

$$\varepsilon_1^* = \frac{a_2^2C_{x_2}^2 + a_2\rho_{yx_2}C_yC_{x_2} - a_1\rho_{yx_1}C_yC_{x_1} - a_1a_2\rho_{x_1x_2}C_{x_1}C_{x_2}}{a_1^2C_{x_1}^2 + a_2^2C_{x_2}^2 - 2a_1a_2\rho_{x_1x_2}C_{x_1}C_{x_2}}$$

$$\theta_1^* = \frac{\left(C_{x_2}^2 + \rho_{yx2}C_yC_{x_2} + \rho_{yx1}C_yC_{x_1} - \rho_{x_1x_2}C_{x_1}C_{x_2}\right)}{C_{x1}^2 + C_{x2}^2 - 2\rho_{x_1x_2}C_{x_1}C_{x_2}}$$

$$a_1^* = \frac{C_yb_{y1.2}}{C_{x_1}}\, a_2^* = \frac{C_yb_{y2.1}}{C_{x_2}},\quad C_1 = \rho_1^* + \rho_{yx_1}C_2 = \rho_2^* + \rho_{yx_2},\quad \alpha_1 = \frac{S_y}{R_1S_{x_1}}\rho_1^*$$

$$\alpha_2 = \frac{S_y}{R_2S_{x_2}}\rho_2^*,\quad \rho_1^* = \frac{\rho_{x_1x_2}\left(\rho_{yx_1}\rho_{x_1x_2} - \rho_{yx_2}\right)}{1-\rho_{x_1x_2}^2}\rho_2^* = \frac{\rho_{x_1x_2}\left(\rho_{yx_2}\rho_{x_1x_2} - \rho_{yx_1}\right)}{1-\rho_{x_1x_2}^2}$$

$$R_1 = \frac{\bar{Y}}{\bar{X}_1},\quad R_2 = \frac{\bar{Y}}{\bar{X}_2},\quad \alpha_{1l} = \frac{a_{1l}\bar{X}_1}{a_1\bar{X}_{1l} + b_{1l}},\quad \alpha_{2l} = \frac{a_{2l}\bar{X}_2}{a_{2l}\bar{X}_2 + b_{2l}}$$

$$R_{y.x_1x_2}^2 = \frac{\rho_{yx_1}^2 + \rho_{yx_2}^2 - 2\rho_{yx1}\rho_{yx_2}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2},\quad \alpha' = \frac{C_y}{C_{x1}}\left[\frac{\rho_{yx_2} - \rho_{yx_1}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2}\right]$$

$$\emptyset opt = \frac{S_y}{S_x}\left[\frac{\rho_{yx1} - \rho_{yx2}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2}\right]$$

$$W_{1l}^* + W_{2l}^* = 1,\quad \bar{x}_{lc} = K_{1l}\bar{x}_1 + K_{2l}\bar{x}_2, \bar{X}_{1C} = K_{1l}\bar{X}_1 + K_{2l}\bar{X}_2$$

$$b_{lc} = \frac{S_{yxlc}}{S_{xlc}^2},\quad K_{1l} + K_{2l} = 1,\quad W_{1l}^* = \frac{\bar{X}_2\left[\propto_l C_{x_2}^2 - \rho_{yx_2}C_yC_{x_2} - \propto_l \rho_{x_1x_2}C_{x_1}C_{x_2} + \rho_{yx_1}C_yC_{x_1}\right]}{\{\bar{X}_1\left[\propto_l C_{x_1}^2 + \rho_{yx_2}C_yC_{x_2} - \propto_l \rho_{x_1x_2}C_{x_1}C_{x_2} - \rho_{yx_1}C_yC_{x_1}\right] + \bar{X}_2\left[\propto_l C_{x_2}^2 - \rho_{yx_2}C_yC_{x_2} - \propto_l \rho_{x_1x_2}C_{x_1}C_{x_2} + \rho_{yx_1}C_yC_{x_1}\right]\}}$$

$$R_{lc} = \frac{\bar{Y}}{W_{1l}^*\bar{X}_1 + W_{2l}^*\bar{X}_2},\quad K_{1l} = \frac{\bar{X}_2C_{x_2}\left(\rho_{yx_1} - \rho_{yx_2}\rho_{x_1x_2}\right)}{\bar{X}_1C_{x_1}\left(\rho_{yx_2} - \rho_{yx_1}\rho_{x_1x_2}\right) + \bar{X}_2C_{x_2}\left(\rho_{yx_1} - \rho_{yx_2}\rho_{x_1x_2}\right)}$$

$$\rho_{yxlc}^2 = \frac{S_{yxlc}}{S_y^2S_{xlc}^2} = \frac{\left[K_{1l}\rho_{yx_1}C_yC_{x_1}\bar{X}_1\bar{Y} + K_{2l}\rho_{yx_2}C_yC_{x_2}\bar{X}_2\bar{Y}\right]^2}{C_y^2\bar{Y}^2\{K_{1l}^2C_{x_1}^2\bar{X}_1^2 + 2K_{1l}K_{2l}\rho_{x_1x_2}C_{x_1}C_{x_2}\bar{X}_1\bar{X}_2 + K_{2l}^2C_{x_2}^2\bar{X}_2^2\}}$$

and

$$w_{1lcr}^{*} = \frac{\acute{B}}{\acute{A}+\acute{B}}, \quad \text{where } \acute{A} = \bar{X}_1^2\bar{X}_2\bar{Y}\left\{2\rho_{yx_2}C_yC_{x_2} - \rho_{x_1x_2}C_{x_1}C_{x_2} - 2\rho_{yx_1}C_yC_{x_1} + C_{x_1}^2\right\}$$

$$\acute{B} = \bar{Y}\bar{X}_1\bar{X}_2^2\left\{C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2} - \rho_{x_1x_2}C_{x_2}C_{x_1} + 2\rho_{yx_1}C_yC_{x_1}\right\}$$

$$R_{1Cr} = \frac{\bar{Y}}{w_{1lcr}^{*}\bar{X}_1 + w_{2lcr}^{*}\bar{X}_2}, \quad w_{1lcr}^{*} + w_{2lcr}^{*} = 1$$

$$\bar{x}_{1t} = (1+g)\bar{X}_1 - g\bar{x}_1, \quad \bar{x}_{2t} = (1+g)\bar{X}_2 - g\bar{x}_2, \quad g = \frac{n}{N-n}$$

$$w_y = 1 - \frac{2\left(2\propto_y gC_yC_{x_1}\rho_{yx_1} + \propto_y gC_yC_{x_2}\rho_{yx_2}\right)}{g^2\left(\propto_y^2 C_{x_2}^2 + 4C_{x_1}^2 + 4\propto_y C_{x_1}C_{x_2}\rho_{x_1x_2}\right)}$$

The proposed classes of multivariate exponential estimators for population mean under simple random sampling. Motivated by Kadilar and Cingi (2004, 2005) Bahl and Tuteja (1991) and Riaz *et al*. (2014), the following exponential estimators are proposed when two auxiliary variables are considered under the simple random sampling scheme.

$$t_{m1} = w_0\bar{y} + w_1(\bar{X}_1 - \bar{x}_1)exp\left[\frac{\delta_1(\bar{X}_1 - \bar{x}_1)}{(\bar{X}_1 + \bar{x}_1)}\right] + w_2(\bar{X}_2 - \bar{x}_2)exp\left[\frac{\delta_2(\bar{X}_2 - \bar{x}_2)}{(\bar{X}_2 + \bar{x}_2)}\right] \quad \dots(1)$$

$$t_{m2} = \varpi_0\bar{y} + \varpi_1(\bar{X}_1 - \bar{x}_1)exp\left[\frac{\delta_2(\bar{X}_2 - \bar{x}_2)}{(\bar{X}_2 + \bar{x}_2)}\right] + \varpi_2(\bar{X}_2 - \bar{x}_2)exp\left[\frac{\delta_1(\bar{X}_1 - \bar{x}_1)}{(\bar{X}_1 + \bar{x}_1)}\right] \quad \dots(2)$$

$$where\ 0 \le \delta_1, \delta_2 \le 2$$

To obtain the bias and mean square error (MSE) of the proposed estimators, (1) and (2) are expressed in terms of the usual $e's$. For t_{m1}, we have

$$t_{m1} = w_0\bar{Y}(1+e_y) - w_1e_{x_1}\bar{X}_1\left[1 - \frac{\delta_1e_{x_1}}{2}\left(1+\frac{\delta_1e_{x_1}}{2}\right)^{-1} + \frac{\delta_1^2e_{x_1}^2}{8}\left(1+\frac{\delta_1e_{x_1}}{2}\right)^{-2} + \cdots\right]$$
$$- w_2e_{x_2}\bar{X}_2\left[1 - \frac{\delta_2e_{x_2}}{2}\left(1+\frac{\delta_2e_{x_2}}{2}\right)^{-1} + \frac{\delta_2^2e_{x_2}^2}{8}\left(1+\frac{\delta_2e_{x_2}}{2}\right)^{-2} + \cdots\right] \quad \dots(3)$$

We assume that,

$$|e_{x_1}| \le 1, |e_{x_2}| \le 1, \left|\frac{\delta_1e_{x_1}}{2}\right| \le 1, \quad \left|\frac{\delta_2e_{x_2}}{2}\right| \le 1$$
$$\Rightarrow |\delta_1| \le 2\ and\ |\delta_2| \le 2$$

Therefore, Expanding (3) to first order approximation gives.

$$t_{m1} = w_0\bar{Y}(1+e_y) - w_1e_{x_1}\bar{X}_1 + \frac{w_1\delta_1e_{x_1}^2\bar{X}_1}{2} - \delta_2w_2e_{x_2}\bar{X}_2 + \frac{\delta_2w_2e_{x_2}^2\bar{X}_2}{2}$$
$$= \bar{Y}\left[w_0 + w_0e_y - w_1M_1e_{x_1} + \frac{w_1\delta_1M_1e_{x_1}^2}{2} - \delta_2w_2M_2e_{x_2} + \frac{\delta_2w_2M_2e_{x_2}^2}{2}\right]$$
$$(t_{m1} - \bar{Y}) = \bar{Y}\left[(w_0-1) + w_0e_y - w_1M_1e_{x_1} + \frac{w_1\delta_1M_1e_{x_1}^2}{2} - \delta_2w_2M_2e_{x_2} + \frac{\delta_2w_2M_2e_{x_2}^2}{2}\right]$$
$$B(t_{m1}) = E(t_{m1} - \bar{Y}) = \bar{Y}\left[(w_0-1) + \frac{w_1\delta_1M_1\lambda C_{x_1}^2}{2} + \frac{\delta_2w_2M_2\lambda C_{x_2}^2}{2}\right] \quad \dots(4)$$

Also,

$$(t_{m1} - \bar{Y})^2 = \bar{Y}^2\left[(w_0-1)^2 + \frac{2(w_0w_1 - w_1)\delta_1M_1e_{x_1}^2}{2} + \frac{2(w_0w_2 - w_2)\delta_2M_2e_{x_2}^2}{2} + w_0^2e_y^2 - 2w_0w_1M_1e_ye_{x_1}\right.$$
$$\left. - 2w_0w_2M_2e_ye_{x_2} + 2w_1w_2M_1M_2e_{x_1}e_{x_2} + w_1^2M_1^2e_{x_1}^2 + w_2^2M_2^2e_{x_2}^2\right]$$

$$MSE(t_{m1}) = \bar{Y}^2\left[1 + w_0^2(1+\lambda C_y^2) - 2w_0 - 2w_0w_1M_1\frac{\lambda C_{x_1}^2}{2}(2K_{yx_1} - \delta_1) - 2w_0w_2M_2\frac{\lambda C_{x_2}^2}{2}(2K_{yx_2} - \delta_2)\right.$$
$$\left. + 2w_1w_2M_1M_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} - \frac{2w_1\delta_1M_1\lambda C_{x_1}^2}{2} - \frac{2w_2\delta_2M_2\lambda C_{x_2}^2}{2} + w_1^2M_1^2\lambda C_{x_1}^2 + w_2^2M_2^2\lambda C_{x_2}^2\right]$$

$$= \bar{Y}^2[1 + w_0^2\mu_1 - 2w_0 - 2w_0w_1M_1\mu_2 - 2w_0w_2M_2\mu_3 + 2w_1w_2M_1M_2\mu_4 - 2w_1M_1\mu_5 - 2w_2M_2\mu_6 + w_1^2M_1^2\mu_7 + w_2^2M_2^2\mu_8] \qquad \ldots(5)$$

where

$$\mu_1 = 1 + \lambda C_y^2, \quad \mu_2 = \frac{\lambda C_{x_1}^2}{2}(2K_{yx_1} - \delta_1), \quad \mu_3 = \frac{\lambda C_{x_2}^2}{2}(2K_{yx_2} - \delta_2), \quad \mu_4 = \lambda\rho_{x_1x_2}C_{x_1}C_{x_2},$$
$$\mu_5 = \frac{\delta_1\lambda C_{x_1}^2}{2}, \quad \mu_6 = \frac{\delta_2\lambda C_{x_2}^2}{2}, \quad \mu_7 = \lambda C_{x_1}^2, \quad \mu_8 = \lambda C_{x_2}^2$$

Considering (2), the bias and the MSE are obtained in a similar way as follows;

$$t_{m2} = \varpi_0\bar{Y}(1 + e_y) - \varpi_1 e_{x_1}\bar{X}_1\left[1 - \frac{\delta_2 e_{x_2}}{2}\left(1 + \frac{e_{x_2}}{2}\right)^{-1} + \frac{\delta_2^2 e_{x_2}^2}{8}\left(1 + \frac{e_{x_2}}{2}\right)^{-2} + \cdots\right]$$
$$- \varpi_2 e_{x_2}\bar{X}_2\left[1 - \frac{\delta_1 e_{x_1}}{2}\left(1 + \frac{e_{x_1}}{2}\right)^{-1} + \frac{\delta_1^2 e_{x_1}^2}{8}\left(1 + \frac{e_{x_1}}{2}\right)^{-2} + \cdots\right]$$
$$= \varpi_0\bar{Y}(1 + e_y) - \varpi_1\bar{X}_1 e_{x_1} + \frac{\varpi_1\bar{X}_1\delta_2 e_{x_2}e_{x_1}}{2} - \varpi_2\bar{X}_2 e_{x_2} + \frac{\varpi_2\bar{X}_2\delta_1 e_{x_2}e_{x_1}}{2}$$
$$= \bar{Y}\left[\varpi_0 + \varpi_0 e_y - \varpi_1 M_1 e_{x_1} - \varpi_2 M_2 e_{x_2} + \frac{e_{x_2}e_{x_1}}{2}(\varpi_1 M_1\delta_2 + \varpi_2 M_2\delta_1)\right]$$

$$(t_{m2} - \bar{Y}) = \bar{Y}\left[(\varpi_0 - 1) + \varpi_0 e_y - \varpi_1 M_1 e_{x_1} - \varpi_2 M_2 e_{x_2} + \frac{e_{x_2}e_{x_1}}{2}(\varpi_1 M_1\delta_2 + \varpi_2 M_2\delta_1)\right] \qquad \ldots\ldots(6)$$

$$B(t_{m2}) = E(t_{m2} - \bar{Y}) = \bar{Y}\left[(\varpi_0 - 1) + \frac{\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}}{2}(\varpi_1 M_1\delta_2 + \varpi_2 M_2\delta_1)\right] \qquad \ldots(7)$$

$$MSE(t_{m2}) = E(t_{m2} - \bar{Y})^2$$
$$= \bar{Y}^2\left[1 + \varpi_0^2(1 + \lambda C_y^2) - 2\varpi_0 - \frac{2\varpi_0\varpi_1 M_1\lambda C_{x_1}^2}{2}(2K_{yx_1} - \delta_2 K_{x_2x_1}) - \frac{2\varpi_0\varpi_2 M_2\lambda C_{x_2}^2}{2}(2K_{yx_2} - \delta_1 K_{x_1x_2}) + 2\varpi_1\varpi_2 M_1 M_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2} - \frac{2\lambda\varpi_1\delta_2 M_1\rho_{x_1x_2}C_{x_1}C_{x_2}}{2} - \frac{2\varpi_2\delta_1 M_2\lambda\rho_{x_1x_2}C_{x_1}C_{x_2}}{2} + \varpi_1^2 M_1^2\lambda C_{x_1}^2 + \varpi_2^2 M_2^2\lambda C_{x_2}^2\right]$$

$$= \bar{Y}^2[1 + \varpi_0^2\mu_1 - 2\varpi_0 - 2\varpi_0\varpi_1 M_1\mu_2' - 2\varpi_0\varpi_2 M_2\mu_3' + 2\varpi_1\varpi_2 M_1 M_2\mu_4' - 2\varpi_1 M_1\mu_5' - 2\varpi_2 M_2\mu_6' + \varpi_1^2 M_1^2\mu_7' + \varpi_2^2 M_2^2\mu_8'] \qquad \ldots(8)$$

$$where\ \mu_1 = 1 + \lambda C_y^2, \quad \mu_2' = \frac{\lambda C_{x_1}^2}{2}(2K_{yx_1} - \delta_2 K_{x_2x_1}), \quad \mu_3' = \frac{\lambda C_{x_2}^2}{2}(2K_{yx_2} - \delta_1 K_{x_1x_2}),$$
$$\mu_4' = \lambda C_{x_1}^2 K_{x_1x_2}, \quad \mu_5' = \frac{\delta_2\lambda C_{x_1}^2 K_{x_2x_1}}{2}, \quad \mu_6' = \frac{\delta_1\lambda C_{x_2}^2 K_{x_1x_2}}{2}, \quad \mu_7' = \lambda C_{x_1}^2, \quad \mu_8' = \lambda C_{x_2}^2$$

Optimality conditions for the proposed estimators For t_{m1}, we obtain the optimality condition by differentiating (5) partially with respect to the parameters w_0, w_1 *and* w_2 and setting the resulting expressions to zero. Thus;

$$\frac{\partial MSE(t_{m1})}{\partial w_0} = 2w_0\mu_1 - 2 - 2w_1M_1\mu_2 - 2w_2M_2\mu_3 = 0$$
$$\Rightarrow w_0\mu_1 - w_1M_1\mu_2 - w_2M_2\mu_3 = 1 \qquad \ldots(9)$$
$$\frac{\partial MSE(t_{m1})}{\partial w_1} = -2w_0M_1\mu_2 + 2w_2M_1M_2\mu_4 - 2w_1\mu_5 + 2w_1M_1^2\mu_7 = 0$$
$$\Rightarrow w_0\mu_2 - w_1M_1\mu_7 - w_2M_2\mu_4 = -\mu_5 \qquad \ldots(10)$$
$$\frac{\partial MSE(t_{m1})}{\partial w_2} = -2w_0M_2\mu_3 + 2w_1M_1M_2\mu_4 - 2M_2\mu_6 + 2w_2M_2^2\mu_8 = 0$$
$$\Rightarrow w_0\mu_3 - w_1M_1\mu_4 - w_2M_2\mu_8 = -\mu_6 \qquad \ldots(11)$$
$$\Rightarrow \begin{pmatrix} \mu_1 & -M_1\mu_2 & -M_2\mu_3 \\ \mu_2 & -M_1\mu_7 & -M_2\mu_4 \\ \mu_3 & -M_1\mu_4 & -M_2\mu_8 \end{pmatrix}\begin{pmatrix} w_0 \\ w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -\mu_5 \\ -\mu_6 \end{pmatrix} \qquad \ldots(12)$$

Solving (12) by Crammer's rule we have;

$$\left.\begin{aligned} w_0 &= \frac{|A_0|}{|A|} = \frac{M_1 M_2 D_0}{M_1 M_2 D} = \frac{D_0}{D} \\ w_1 &= \frac{|A_1|}{|A|} = \frac{M_2 D_1}{M_1 M_2 D} = \frac{D_1}{M_1 D} \\ w_2 &= \frac{|A_2|}{|A|} = \frac{M_1 D_2}{M_1 M_2 D} = \frac{D_2}{M_2 D} \end{aligned}\right\} \qquad \dots(13)$$

where

$D = \mu_1\mu_7\mu_8 - \mu_1\mu_4^2 - \mu_2^2\mu_8 + 2\mu_2\mu_3\mu_4 - \mu_3^2\mu_7$

$D_0 = \mu_7\mu_8 - \mu_4^2 + \mu_2\mu_5\mu_8 - \mu_3\mu_4\mu_5 - \mu_2\mu_4\mu_6 + \mu_3\mu_6\mu_7$

$D_1 = \mu_1\mu_5\mu_8 - \mu_1\mu_4\mu_6 + \mu_2\mu_8 - \mu_3\mu_4 + \mu_2\mu_3\mu_6 - \mu_3^2\mu_5$

$D_2 = \mu_1\mu_6\mu_7 - \mu_1\mu_4\mu_5 - \mu_2^2\mu_6 + \mu_2\mu_3\mu_5 - \mu_2\mu_4 + \mu_3\mu_7$

Substituting (13) in (5), gives the optimum MSE of t_{m1} as

$$MSE_{opt}(t_{m1}) = \bar{Y}^2\left(1 + \frac{D_0^2\mu_1}{D^2} - \frac{2D_0D}{D^2} - \frac{2D_0D_1\mu_2}{D^2} - \frac{2D_0D_2\mu_3}{D^2} + \frac{2D_1D_2\mu_4}{D^2} - \frac{2D_1D\mu_5}{D^2} - \frac{2D_2D\mu_6}{D^2} + \frac{D_1^2\mu_7}{D^2} + \frac{D_2^2\mu_8}{D^2}\right)$$

$$= \bar{Y}^2\left[1 + \frac{1}{D^2}\left(D_0^2\mu_1 - 2D_0D - 2D_0D_1\mu_2 - 2D_0D_2\mu_3 + 2D_1D_2\mu_4 - 2D_1D\mu_5 - 2D_2D\mu_6 + D_1^2\mu_7 + D_2^2\mu_8\right)\right]$$

$$= \bar{Y}^2\left(1 + \frac{P}{Q}\right) \qquad \dots(14)$$

where

$P = D_0^2\mu_1 - 2D_0D - 2D_0D_1\mu_2 - 2D_0D_2\mu_3 + 2D_1D_2\mu_4 - 2D_1D\mu_5 - 2D_2D\mu_6 + D_1^2\mu_7 + D_2^2\mu_8$

$Q = D^2$

For the second proposed estimator t_{m2}, the optimality condition follows a similar procedure as that of t_{m1}. Again, (8) is differentiated partially with respect to ϖ_0, ϖ_1 and ϖ_2 and setting the resulting expressions to zero. These resulting equations are solved simultaneously to obtain the values of ϖ_0, ϖ_1 and ϖ_2. Thus;

$$\frac{\partial MSE(t_{m2})}{\partial \varpi_0} = 2\varpi_0\mu_1 - 2 - 2\varpi_1 M_1\mu_2' - 2\varpi_2 M_2\mu_3' = 0$$

$$\Rightarrow \varpi_0\mu_1 - \varpi_1 M_1\mu_2' - \varpi_2 M_2\mu_3' = 1 \qquad \dots(15)$$

$$\frac{\partial MSE(t_{m2})}{\partial \varpi_1} = -2\varpi_0 M_1\mu_2' + 2\varpi_2 M_1 M_2\mu_4 - M_1\mu_5' + 2\varpi_1 M_1^2\mu_7 = 0$$

$$\Rightarrow \varpi_0\mu_2' - \varpi_1 M_1\mu_7 - \varpi_2 M_2\mu_4 = -\mu_5' \qquad \dots(16)$$

$$\frac{\partial MSE(t_{m2})}{\partial \varpi_2} = -2\varpi_0 M_2\mu_3' + 2\varpi_1 M_1 M_2\mu_4 - M_2\mu_6' + 2\varpi_2 M_2^2\mu_8 = 0$$

$$\Rightarrow \varpi_0\mu_3' - \varpi_1 M_1\mu_4 - \varpi_2 M_2\mu_8 = -\mu_6' \qquad \dots(17)$$

$$\Rightarrow \begin{pmatrix} \mu_1 & -M_1\mu_2' & M_2\mu_3' \\ \mu_2' & -M_1\mu_7 & M_2\mu_4 \\ \mu_3' & -M_1\mu_4 & M_2\mu_8 \end{pmatrix}\begin{pmatrix} \varpi_0 \\ \varpi_1 \\ \varpi_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \mu_5' \\ \mu_6' \end{pmatrix} \qquad \dots(18)$$

Solving (18) simultaneously gives the following results;

$$D' = \mu_1\mu_7\mu_8 - \mu_1\mu_4^2 - \mu_2'^2\mu_8 + 2\mu_2'\mu_3'\mu_4' - \mu_3'^2\mu_7$$
$$D_0' = \mu_7\mu_8 - \mu_4^2 + \mu_2'\mu_5'\mu_8 - \mu_2'\mu_3'\mu_4 - \mu_2'\mu_4\mu_6' + \mu_3'\mu_6'\mu_7$$
$$D_1' = \mu_1\mu_5'\mu_8 - \mu_1\mu_4\mu_6' + \mu_2'\mu_8 - \mu_3'\mu_4 + \mu_2'\mu_3'\mu_6' - \mu_3'^2\mu_6'$$
$$D_2' = \mu_1\mu_6'\mu_7 - \mu_1\mu_4\mu_5' - \mu_2'^2\mu_6' + \mu_2'\mu_3'\mu_5' - \mu_2'\mu_4 + \mu_3'\mu_7$$

$$\left.\therefore \varpi_0 = \frac{D_0'}{D'}, \qquad \varpi_1 = \frac{D_1'}{D'}, \qquad \varpi_2 = \frac{D_2'}{D'}\right\} \qquad \dots(19)$$

Substituting (19) in (8), gives the optimum MSE of t_{m2} as

$$MSE_{opt}(t_{m2}) = \bar{Y}^2\left(1 + \frac{P'}{Q'}\right) \qquad \dots(20)$$

where

$P' = D_0'^2\mu_1 - 2D'D_0' - 2D_0'D_1'\mu_2' - 2D_0'D_2'\mu_3' + 2D_1'D_2'\mu_4' - 2D'D_1'\mu_5' - 2D'D_2'\mu_6' + D_1'^2\mu_7 + D_2'^2\mu_8$

$$Q' = D'^2$$

Some members of the proposed classes of multivariate ratio-type estimators of Population mean in Simple Random Sampling are shown below:

(a) Some members of t_{m1}

Varying the values of $\delta_1, \delta_2, w_0, w_1$ *and* w_2 gives some ratio-type exponential estimators using two auxiliary variables as shown in Table 2.

Table 2: Some Members of the proposed multivariate estimator (t_{m1}) of population mean in simple random sampling

Estimator	$\boldsymbol{\delta_1}$	$\boldsymbol{\delta_2}$	w_0	w_1	w_2
$t_{m1}^{(1)} = \bar{y}$, *Sample mean*	δ_1	δ_2	1	0	0
$t_{m1}^{(2)} = \bar{y} + b_{yx_1}(\bar{X}_1 - \bar{x}_1) + b_{yx_2}(\bar{X}_2 - \bar{x}_2)$	0	0	1	b_{yx_1}	b_{yx_2}
$t_{m1}^{(3)} = \bar{y} + b_{yx_1}(\bar{X}_1 - \bar{x}_1)exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right) + b_{yx_2}(\bar{X}_2 - \bar{x}_2)exp\left(\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right)$	1	1	1	b_{yx_1}	b_{yx_2}
$t_{m1}^{(4)} = \bar{y} + w_{11}^*(\bar{X}_1 - \bar{x}_1)exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right) + w_{21}^*(\bar{X}_2 - \bar{x}_2)exp\left(\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right)$	1	1	1	w_{11}^*	w_{21}^*
$t_{m1}^{(5)} = \bar{y} + b_{yx_1}(\bar{X}_1 - \bar{x}_1)exp\left[\frac{2(\bar{X}_1 - \bar{x}_1)}{\bar{X}_1 + \bar{x}_1}\right] + b_{yx_2}(\bar{X}_2 - \bar{x}_2)exp\left[\frac{2(\bar{X}_2 - \bar{x}_2)}{\bar{X}_2 + \bar{x}_2}\right]$	2	2	1	b_{yx_1}	b_{yx_2}
$t_{m1}^{(6)} = \bar{y} + w_{12}^*(\bar{X}_1 - \bar{x}_1)exp\left[\frac{2(\bar{X}_1 - \bar{x}_1)}{\bar{X}_1 + \bar{x}_1}\right] + w_{22}^*(\bar{X}_2 - \bar{x}_2)exp\left[\frac{2(\bar{X}_2 - \bar{x}_2)}{\bar{X}_2 + \bar{x}_2}\right]$	2	2	1	w_{12}^*	w_{22}^*
$t_{m1}^{(7)} = w_0^*\bar{y} + w_1^*(\bar{X}_1 - \bar{x}_1)exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right) + w_2^*(\bar{X}_2 - \bar{x}_2)exp\left(\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right)$	1	1	w_0^*	w_1^*	w_2^*
$t_{m1}^{(8)} = w_0^{*\prime}\bar{y} + w_1^{*\prime}(\bar{X}_1 - \bar{x}_1)exp\left[\frac{2(\bar{X}_1 - \bar{x}_1)}{\bar{X}_1 + \bar{x}_1}\right] + w_2^{*\prime}(\bar{X}_2 - \bar{x}_2)exp\left[\frac{2(\bar{X}_2 - \bar{x}_2)}{\bar{X}_2 + \bar{x}_2}\right]$	2	2	$w_0^{*\prime}$	$w_1^{*\prime}$	$w_2^{*\prime}$

(b) Some members of t_{m2} Varying the values of $\delta_1, \delta_2, \varpi_0, \varpi_1$ *and* ϖ_2 gives some ratio-type exponential estimators using two auxiliary variables as shown in Table 3 below.

Table 3: Some Members of the proposed multivariate estimator(t_{m2}) of the population mean in simple random sampling

Estimator	$\boldsymbol{\delta_1}$	$\boldsymbol{\delta_2}$	$\boldsymbol{\varpi_0}$	$\boldsymbol{\varpi_1}$	$\boldsymbol{\varpi_2}$
$t_{m2}^{(1)} = \bar{y}$, *Sample mean*	δ_1	δ_2	1	0	0
$t_{m2}^{(2)} = \bar{y} + b_{yx_1}(\bar{X}_1 - \bar{x}_1) + b_{yx_2}(\bar{X}_2 - \bar{x}_2)$	0	0	1	b_{yx_1}	b_{yx_2}

$t_{m2}^{(3)} = \bar{y} + b_{yx_1}(\bar{X}_1 - \bar{x}_1)exp\left(\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right) + b_{yx_2}(\bar{X}_2 - \bar{x}_2)exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right)$	1	1	1	b_{yx_1}	b_{yx_2}
$t_{m2}^{(4)} = \bar{y} + \varpi_{11}^*(\bar{X}_1 - \bar{x}_1)exp\left(\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right) + \varpi_{21}^*(\bar{X}_2 - \bar{x}_2)exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right)$	1	1	1	ϖ_{11}^*	ϖ_{21}^*
$t_{m2}^{(5)} = \bar{y} + b_{yx_1}(\bar{X}_1 - \bar{x}_1)exp\left[\frac{2(\bar{X}_2 - \bar{x}_2)}{\bar{X}_2 + \bar{x}_2}\right] + b_{yx_2}(\bar{X}_2 - \bar{x}_2)exp\left[\frac{2(\bar{X}_1 - \bar{x}_1)}{\bar{X}_1 + \bar{x}_1}\right]$	2	2	1	b_{yx_1}	b_{yx_2}
$t_{m2}^{(6)} = \bar{y} + \varpi_{12}^*(\bar{X}_1 - \bar{x}_1)exp\left[\frac{2(\bar{X}_2 - \bar{x}_2)}{\bar{X}_2 + \bar{x}_2}\right] + \varpi_{22}^*(\bar{X}_2 - \bar{x}_2)exp\left[\frac{2(\bar{X}_1 - \bar{x}_1)}{\bar{X}_1 + \bar{x}_1}\right]$	2	2	1	ϖ_{12}^*	ϖ_{22}^*
$t_{m2}^{(7)} = \varpi_0^*\bar{y} + \varpi_1^*(\bar{X}_1 - \bar{x}_1)exp\left(\frac{\bar{X}_2 - \bar{x}_2}{\bar{X}_2 + \bar{x}_2}\right) + \varpi_2^*(\bar{X}_2 - \bar{x}_2)exp\left(\frac{\bar{X}_1 - \bar{x}_1}{\bar{X}_1 + \bar{x}_1}\right)$	1	1	ϖ_0^*	ϖ_1^*	ϖ_2^*
$t_{m2}^{(8)} = \varpi_0^{*\prime}\bar{y} + \varpi_1^{*\prime}(\bar{X}_1 - \bar{x}_1)exp\left[\frac{2(\bar{X}_2 - \bar{x}_2)}{\bar{X}_2 + \bar{x}_2}\right] + \varpi_2^{*\prime}(\bar{X}_2 - \bar{x}_2)exp\left[\frac{2(\bar{X}_1 - \bar{x}_1)}{\bar{X}_1 + \bar{x}_1}\right]$	2	2	$\varpi_0^{*\prime}$	$\varpi_1^{*\prime}$	$\varpi_2^{*\prime}$

Efficiency comparison

The efficiency of the proposed estimators are compared with other relevant existing estimators and members of the family of generalized estimators using the percent relative efficiency given as:

$$PRE = \frac{MSE(\bar{y}_{rmi})}{MSE(t_{mi})} \times 100$$

where $\bar{y}_{rmi}$ is the estimator to be compared with and t_{mi} is the proposed estimator.

The efficiencies of the proposed estimators of population mean using two auxilliary variable in simple random sampling can also be compared by establishing conditions under which the proposed estimators would be more efficient than the existing ones; Here we consider the best among the class of proposed estimators.

(a) Comparison with classical multivariate ratio estimator

The proposed estimator is more efficient than the classical multivariate ratio estimator if;

$$MSE_{min}(t_{mi}) \leq MSE_{min}(\bar{y}_{mR})$$

$$\Rightarrow \bar{Y}^2\left(1 + \frac{P_i}{Q_i}\right) \leq \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + \theta_1^{*^2}C_{x_1}^2 + \theta_2^{*^2}C_{x_2}^2 - 2\theta_1^*\rho_{yx_1}C_yC_{x_1} - 2\theta_2^*\rho_{yx_2}C_yC_{x_2} + 2\theta_1^*\theta_2^*\rho_{x_1x_2}C_{x_1}C_{x_2}\right)$$

$$\Rightarrow \bar{Y}^2\left(1 + \frac{P_i}{Q_i}\right) \leq \lambda\bar{Y}^2A_1$$

$$\Rightarrow 1 + \frac{P_i}{Q_i} - \lambda A_1 \leq 0 \qquad \dots(21)$$

$$where\ A_1 = C_y^2 + \theta_1^{*^2}C_{x_1}^2 + \theta_2^{*^2}C_{x_2}^2 - 2\theta_1^*\rho_{yx_1}C_yC_{x_1} - 2\theta_2^*\rho_{yx_2}C_yC_{x_2} + 2\theta_1^*\theta_2^*\rho_{x_1x_2}C_{x_1}C_{x_2}$$

When condition (21) holds, then the first proposed class of estimators is more efficient than the traditional ratio estimator proposed by Singh and Chaudhary (1986). Similarly, for the second proposed estimator, the condition is;

$$1 + \frac{P_i'}{Q_i'} - \lambda A_1 \leq 0 \qquad \dots(22)$$

(b) Comparison with Abu-Dayeh *et al*. (2003)

The proposed class of estimators would be more efficient than the Abu-Dayeh *et al.* (2003) estimator if the following holds:

(i) $MSE_{min}(t_{m1}) \leq MSE_{min}(\bar{y}_{r2}^r)$

$$\Rightarrow \bar{Y}^2\left(1+\frac{P}{Q}\right) \leq \lambda \bar{Y}^2 C_y^2\left(1-\frac{\rho_{yx_1}^2+\rho_{yx_2}^2-2\rho_{yx_1}\rho_{yx_2}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2}\right)$$

$$\Rightarrow 1+\frac{P}{Q}-\lambda C_y^2\left(1-R_{yx_1x_2}\right) \leq 0 \quad \ldots(23)$$

$where\ R_{y.x_1x_2}^2=\dfrac{\rho_{yx_1}^2+\rho_{yx_2}^2-2\rho_{yx_1}\rho_{yx_2}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2}$

(ii) For the second proposed estimator of Abu-Dayeh *et al.* (2003), the condition becomes

$$\bar{Y}^2\left(1+\frac{P}{Q}\right) \leq MSE_{min}(\bar{y}_{r2}^{\varepsilon})$$

$$\Rightarrow \bar{Y}^2\left(1+\frac{P}{Q}\right) \leq \frac{1-f}{n}\bar{Y}^2\left(C_y^2+a_1^{*^2}r_1^2C_{x_1}^2+a_2^{*^2}r_2^2C_{x_2}^2-2a_1^*r_1\rho_{yx_1}C_yC_{x_1}-2a_2^*r_2\rho_{yx_2}C_yC_{x_2}+2a_1^*a_2^*r_1r_2\rho_{x_1x_2}C_{x_1}C_{x_2}\right)$$

$$\Rightarrow 1+\frac{P}{Q}-\lambda B_1 \leq 0 \quad \ldots(24)$$

where

$$B_1=C_y^2+a_1^{*^2}r_1^2C_{x_1}^2+a_2^{*^2}r_2^2C_{x_2}^2-2a_1^*r_1\rho_{yx_1}C_yC_{x_1}-2a_2^*r_2\rho_{yx_2}C_yC_{x_2}+2a_1^*a_2^*r_1r_2\rho_{x_1x_2}C_{x_1}C_{x_2}$$

The second proposed class of estimators is more efficient than the Abu-Dayeh *et al* .(2003) estimator if;

$$1+\frac{P'}{Q'}-\lambda R_1 \leq 0 \quad \ldots(25)$$

$$and \qquad 1+\frac{P'}{Q'}-\lambda B_1 \leq 0 \quad \ldots(26)$$

(c) Comparison with Lu and Yan (2014)

For the proposed class of estimators to be more efficient the the Lu and Yan (2014) estimator

$$MSE_{min}(t_{mi}) \leq MSE_{min}(\bar{y}_{LY})$$

(i) $MSE_{min}(t_{m1}) \leq MSE_{min}(\bar{y}_{LY})$

$$\Rightarrow \bar{Y}^2\left(1+\frac{P}{Q}\right) \leq \lambda \bar{Y}^2\left(C_y^2+W_{1l}^{*^2}\alpha_{1l}^2C_{x_1}^2+W_{2l}^{*^2}\alpha_{2l}^2C_{x_2}^2-2W_{1l}^*\alpha_{1l}\rho_{yx_1}C_yC_{x_1}-2W_{2l}^*\alpha_{2l}\rho_{yx_2}C_yC_{x_2}+2W_{1l}^*W_{2l}^*\alpha_{1l}\alpha_{2l}\rho_{x_1x_2}C_{x_1}C_{x_2}\right)$$

$$\Rightarrow 1+\frac{P}{Q}-\lambda C_1 \leq 0 \quad \ldots(27)$$

where

$$C_1=C_y^2+W_{1l}^{*^2}\alpha_{1l}^2C_{x_1}^2+W_{2l}^{*^2}\alpha_{2l}^2C_{x_2}^2-2W_{1l}^*\alpha_{1l}\rho_{yx_1}C_yC_{x_1}-2W_{2l}^*\alpha_{2l}\rho_{yx_2}C_yC_{x_2}+2W_{1l}^*W_{2l}^*\alpha_{1l}\alpha_{2l}\rho_{x_1x_2}C_{x_1}C_{x_2}$$

(ii) For the second proposed class of estimators, the condition is

$$1+\frac{P'}{Q'}-\lambda C_1 \leq 0 \quad \ldots(28)$$

(d) Comparison with Swain (2012)

The proposed class of estimators are more efficient than Swain (2012)estimator if;

$$MSE_{min}(t_{mi}) \leq MSE_{min}(\bar{y}_S)$$

(i) $MSE_{min}(t_{m1}) \leq MSE_{min}(\bar{y}_S)$

$$\Rightarrow \bar{Y}^2\left(1+\frac{P}{Q}\right) \leq \lambda \bar{Y}^2C_y^2\left(1-R_{y.x_1x_2}^2\right)$$

$$\Rightarrow 1+\frac{P}{Q}-\lambda C_y^2\left(1-R_{y.x_1x_2}^2\right) \leq 0 \quad \ldots(29)$$

(ii) $MSE_{min}(t_{m2}) \leq MSE_{min}(\bar{y}_S)$

$$\Rightarrow \bar{Y}^2\left(1+\frac{P'}{Q'}\right) \leq \lambda \bar{Y}^2C_y^2\left(1-R_{y.x_1x_2}^2\right)$$

$$\Rightarrow 1+\frac{P'}{Q'}-\lambda C_y^2\left(1-R_{y.x_1x_2}^2\right) \leq 0 \quad \ldots(30)$$

When (29) and (30) hold, then the proposed estimators are more efficient the Swain (2012) estimators.

(e) Comparison among the proposed estimators

The proposed estimator t_{mi} is more efficient than another proposed estimator t_{mj} if;

$$MSE_{min}(t_{mi}) \leq MSE_{min}(t_{mj})$$

(i) $MSE_{min}(t_{m1}) \leq MSE_{min}(t_{m2})$

$$\Rightarrow \bar{Y}^2\left(1+\frac{P}{Q}\right) \leq \bar{Y}^2\left(1+\frac{P'}{Q'}\right)$$

$$\Rightarrow 1+\frac{P}{Q}-1-\frac{P'}{Q'} \leq 0$$

$$\Rightarrow \frac{P}{Q}-\frac{P'}{Q'} \leq 0 \qquad \ldots(31)$$

Numerical Illustration

The Mean Squared Errors (MSE) and Percent Relative Efficiencies (PRE) of the proposed classes of estimators and already existing estimators are presented in Tables 4, 5 and 6. These tables will enable the proposed estimators to be compared with the already existing estimators that are relevant to this work.

Table 4: Data statistics for numerical Illustration of the efficiency of the proposed estimators of population mean in simple random sampling involving two auxiliary variables

Source	N	n	$\bar{Y}$	$\bar{X}_1$	$\bar{X}_2$	C_y	C_{x_1}	C_{x_2}	ρ_{yx_1}	ρ_{yx_2}	$\rho_{x_1x_2}$
Abu-Dayehh (2003), (I)	332	80	1093.1	181.57	143.31	0.7626	0.7684	0.7616	0.973	0.862	0.842
Rashid *et al.* (2014) (II)	424	169	646.215	4533.981	325.0325	1.509	1.342	1.335	0.623	0.907	0.682
Kadilar & Cingi (2005) (III)	204	50	966	26441	1014	2.4737	1.7171	2.4866	0.71	0.94	0.83
Anderson (1958) (IV)	25	15	183.84	185.72	151.12	0.0546	0.0488	0.0546	0.6932	0.7108	0.7346
Kadilar & Cingi (2004) (V)	200	50	500	2500	3000	2	0.2	0.25	0.9	0.85	0.8
Feng & Shi (1996) (VI)	180	70	13.9951	27.3981	38.7167	0.418	0.4254	0.3339	0.563	0.5273	0.2589
Singh & Singh (2014) (VII)	200	70	500	2500	3000	2	0.2	0.2	0.9	0.85	0.8
Singh & Chaudhary (1986) (VIII)	34	20	856.41	208.88	199.44	0.86	0.72	0.75	0.45	0.45	0.98
Choudhary & Singh (2012) (IX)	34	10	4.92	2.99	2.91	1.012	1.232	1.072	0.733	0.643	0.684
Ahmed (1997) (X)	376	159	316.65	141.13	1075.31	0.7721	0.845	0.7746	0.9106	0.9094	0.8614

Table 4 shows parameters of the ten populations that are used to compute the mean square errors of the proposed estimators and some related existing estimators for efficiency comparison.

Table 5: MSE of related existing estimators of Population Mean in simple random sampling using two auxiliary variables

	Population

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Estimators	I	II	III	IV	V	VI	VII	VIII	IX	X
Singh & Chaudhary (1986)	310.461	601.791	10290.66	1.2022	11745.83	0.1577	7686.25	10580.57	0.9055	24.867
PRE	2123.63	562.31	837.77	223.49	127.71	189.49	120.81	105.55	193.26	872.49
Regression	4764.407	1894.88	62088.67	1.9836	10372.50	0.1669	6421.071	11077.64	1.2145	167.757
PRE	138.38	178.58	138.85	135.47	144.61	179.03	144.61	100.82	144.09	129.79
Kadilar & Cingi (2004)	5665.958	2381.8	64002.95	2.7864	2150.34	0.2200	1328.980	16730.74	2.2404	216.480
PRE	116.36	142.07	134.70	96.43	697.56	135.81	698.71	66.75	78.11	100.22
Kadilar & Cingi (2005)	309.848	600.018	8669.442	1.1586	2145.833	0.1575	1328.373	8883.72	0.7438	23.917
PRE	2127.83	563.97	994.44	231.91	699.03	189.75	699.03	125.71	235.29	907.16
Abu-Dayyeh *etal* (2003), $\bar{y}_a$	309.848	600.018	8669.442	1.1586	2145.833	0.1575	1328.373	8883.72	0.7438	23.917
PRE	2127.83	563.97	994.44	231.91	699.03	189.75	699.03	125.71	235.29	907.16
Abu-Dayyeh *etal* (2003), $\bar{y}_w$	365.847	600.143	9664.566	1.5193	3890.033	0.1923	2408.116	9454.816	0.8590	72.119
PRE	1802.13	563.85	892.04	176.84	385.60	155.35	385.60	118.12	203.73	300.84
Lu (2013), $\bar{y}_{\alpha lcr}$	315.339	610.414	9716.309	1.1816	11842.07	0.1575	7744.698	10302.86	0.8647	24.066
PRE	2090.78	554.37	887.29	227.38	126.67	189.75	119.90	108.40	202.37	901.51
Lu (2013), $\bar{y}_{lcreg}$	309.848	600.018	8669.442	1.1586	2145.833	0.1575	1328.373	8883.72	0.7438	23.917
PRE	2127.83	563.97	994.44	231.91	699.03	189.75	699.03	125.71	235.29	907.16
Lu & Yan (2014), $\bar{y}_{pmr}^{(10)}$	309.854	602.038	10280.49	1.1981	11743.96	0.1575	7687.873	10519.14	0.7447	24.696
PRE	2127.79	562.08	838.60	224.25	127.73	189.75	120.78	106.17	234.99	878.53
Lu, Yan & Peng (2014), $\bar{y}_{lcr}$	1776.465	1224.55	12103.28	1.4129	12983.33	0.1887	8436.071	8891.691	0.8203	64.499
PRE	371.13	276.34	712.31	190.16	115.53	158.32	110.07	125.60	213.32	336.38
Riaz *et al* (2014)	309.848	600.018	8669.44	1.1586	2145.833	0.1575	1328.373	8883.72	0.7438	23.917
PRE	2127.83	563.97	994.44	231.91	699.03	189.75	699.03	125.71	235.29	907.16
Singh & Singh (2014)	309.461	600.018	8669.44	1.1586	2145.833	0.1575	1328.373	8883.72	0.7438	23.917
PRE	2127.83	563.97	994.44	231.91	699.03	189.75	699.03	125.71	235.29	907.16
Swain (2012)	309.848	600.018	8669.442	1.1586	2145.833	0.1575	1328.373	8883.72	0.7438	23.917
PRE	2127.83	563.97	994.44	231.91	699.03	189.75	699.03	125.71	235.29	907.16
Yasmeen *et al .* (2015)	379.147	1356.63	25058.02	1.1890	2146.814	0.1683	1333.406	8886.008	0.7441	25.787
PRE	1738.92	249.44	344.05	225.93	698.71	177.55	696.39	125.68	235.17	841.36

Note: Figures in bold are Percent Relative Efficiencies of each estimator in all the populations
Table 2 gives the mean square errors of recent and related estimators of population mean in simple random sampling using two auxiliary variables, proposed by the stated authors.

Table 6: MSE and PRE of the proposed class of Estimator, t_{m1} of population mean in simple random sampling using two auxiliary variables

	Population									
t_{m1}	I	II	III	IV	V	VI	VII	VIII	IX	X

$t_{m1}^{(1)}$	6593.042	3383.921	86212.23	2.6868	15000	0.2988	9285.714	11168.105	1.7500	216.961
PRE	100	100	100	100	100	100	100	100	100	100
$t_{m1}^{(2)}$	4764.407	1894.878	62088.67	1.9833	10372.5	0.1669	6421.071	11077.64	1.2145	167.157
PRE	138.38	178.58	138.85	135.47	144.61	179.03	144.61	100.82	144.09	129.79
$t_{m1}^{(3)}$	4764.407	1894.878	62088.67	1.9833	10372.5	0.1669	6421.071	11077.64	1.2145	167.157
PRE	138.38	178.58	138.85	135.47	144.61	179.03	144.61	100.82	144.09	129.79
$t_{m1}^{(4)}$	309.861	600.036	8810.397	1.1586	2148.33	0.1575	1328.726	8883.740	0.7439	23.917
PRE	2127.74	563.95	978.53	231.91	698.22	189.75	698.84	125.71	235.25	907.17
$t_{m1}^{(5)}$	4764.407	1894.878	62088.67	1.9833	10372.5	0.1669	6421.071	11077.64	1.2145	167.157
PRE	138.38	178.58	138.85	135.47	144.61	179.03	144.61	100.82	144.09	129.79
$t_{m1}^{(6)}$	309.857	600.049	8684.083	1.1586	2150.01	0.1575	1328.859	8883.930	0.7488	23.917
PRE	2127.77	563.94	992.76	231.91	697.67	189.75	698.77	125.71	233.71	907.16
$t_{m1}^{(7)}$	298.961	590.858	6134.77	1.1584	2111.82	0.1571	1315.814	8719.628	0.6478	23.735
PRE	2205.32	572.71	1405.31	231.93	710.29	190.22	705.70	128.08	270.14	914.11
$t_{m1}^{(8)}$	269.992*	573.751*	196.274*	1.1583*	2091.36*	0.1567*	1308.699*	8648.494*	0.5161*	23.312*
PRE	2441.94*	589.79*	43924.52*	231.97*	717.24*	190.71*	709.54*	129.13*	339.07*	930.69*

Note: (1) Percent Relative Efficiencies in bold
(2) * Largest PRE and the least MSE

Table 6 indicates some members of the first proposed class of estimators of population mean in simple random sampling involving two auxiliary variables.

Table 7: MSE of the proposed class of Estimator, t_{m2} of population mean in simple random sampling using two auxiliary variables

	Population									
t_{m2}	I	II	III	IV	V	VI	VII	VIII	IX	X
$t_{m2}^{(1)}$	6593.042	3383.921	86212.23	2.6868	15000	0.2988	9285.714	11168.105	1.7500	216.961
PRE	100	100	100	100	100	100	100	100	100	100
$t_{m2}^{(2)}$	4764.407	1894.878	62088.67	1.9833	10372.5	0.1669	6421.071	11077.64	1.2145	167.157
PRE	138.38	178.58	138.85	135.47	144.61	179.03	144.61	100.82	144.09	129.79
$t_{m2}^{(3)}$	4764.407	1894.878	62088.67	1.9833	10372.5	0.1669	6421.071	11077.64	1.2145	167.157
PRE	138.38	178.58	138.85	135.47	144.61	179.03	144.61	100.82	144.09	129.79
$t_{m2}^{(4)}$	309.861	600.036	8716.403	1.1586	2147.26	0.1575	1328.699	8747.485*	0.6793	23.776
PRE	2127.74	563.95	989.08	231.91	698.56	189.75	698.86	127.67*	257.62	912.52
$t_{m2}^{(5)}$	4764.407	1894.878	62088.67	1.9833	10372.5	0.1669	6421.071	11077.64	1.2145	167.157
PRE	138.38	178.58	138.85	135.47	144.61	179.03	144.61	100.82	144.09	129.79
$t_{m2}^{(6)}$	309.856	600.036	8827.483	1.1586	2147.78	0.1575	1328.806	8884.352	0.7446	23.917
PRE	2127.77	563.95	976.63	231.91	698.40	189.75	698.80	125.70	235.03	907.16
$t_{m2}^{(7)}$	302.981	594.421	8205.277*	1.1585	4039.04	0.1573	2083.815	8747.485*	0.6794	23.776
PRE	2176.06	569.28	1050.69*	231.92	366.48	189.96	445.61	127.67*	257.58	912.52
$t_{m2}^{(8)}$	281.480*	586.908*	13375.96	1.1584*	3416.52	0.1572*	1873.034	31100.99	0.6178*	23.458*
PRE	2342.28*	576.57*	644.53	231.94*	439.04	190.04*	495.76	35.91	283.26*	924.89*

Note: (1) Figures in bold are Percent Relative Efficiencies. (2) * indicates the least MSE and greatest PRE in each population.

Discussion

The optimal MSE of the two multivariate ratio estimators of population mean in simple random sampling scheme using two auxiliary variables proposed in equations (1) and (2) respectively shown in equations (5) and (8) are dependent upon the parameters of the estimators (δ_1, δ_2). Tables 9 and 10 show Asymptotic Optimal Estimators (AOE's) of these proposed multivariate estimators at various values of δ_1 and δ_2, in the presence of two auxiliary variables.

From Table 5, it is seen that estimators of Abu-Dayyeh *et al.* (2003), $\bar{y}_a$, Kadilar and Cingi (2005), Lu (2013), $\bar{y}_{lcreg}$, Riaz *et al* . 2014), Singh and Singh (2014) and Swain (2012) have the same and the least Mean Square Errors in all the populations, among the considered estimators. These are followed by the most efficient member of the class of estimators proposed by Lu and Yan (2014), $\bar{y}_{pmr}^{(10)}$, which has smaller mean square errors in six of the ten populations considered in this work-I, II, III,V, IX and X. The estimator of Singh and Chaudhary (1986) is the usual traditional ratio estimator involving two auxiliary variables. In Table 7, estimator $t_{m_2}^{(1)}$ is the simple random sample mean, on which the efficiencies of other estimators are based. Estimators $t_{m_2}^{(2)}, t_{m_2}^{(3)}$ and $t_{m_2}^{(5)}$ have the same mean square errors as the multivariate regression estimator, when two auxiliary variables are involved. Numerical results in Tables 6 and 7 indicate the mean square errors and PRE'S of existing and the proposed multivariate estimators of the population mean computed on the ten (10) different populations. In the Table, estimators $t_{m_1}^{(2)}$, $t_{m_1}^{(3)}$ and $t_{m_1}^{(6)}$ have the same MSE as the multivariate regression estimator involving two variables, while $t_{m_1}^{(1)}$ is the simple random sample mean. It is also observed that $t_{m_1}^{(4)}$, $t_{m_1}^{(5)}$ and $t_{m_1}^{(7)}$ have almost the same MSEs as the estimators of Abu-Dayyeh (2003), $\bar{y}_a$, Kadilar and Cingi (2005), Lu (2013), $\bar{y}_{lcreg}$, Riaz *et al.* (2014), Singh and Singh (2014) and Swain (2012), except for a slight difference in population III. Also, in Table 6, estimators $t_{m_1}^{(7)}$ and $t_{m_1}^{(8)}$ have less MSEs and greater PREs than all existing estimators considered here, but $t_{m_1}^{(8)}$ gives the least MSE and greatest PRE in all the populations. This indicates that the last three members of the first proposed estimator are more efficient than the already existing ones and the most efficient of all of them is $t_{m_1}^{(8)}$. It is to be noted here that their efficiencies are consistent in all the populations. In Table 7, $t_{m_2}^{(8)}$ has least mean square errors in populations I, II, IV, VII, IX and X and $t_{m_2}^{(7)}$ has the least MSE in populations III and VIII. Estimator $t_{m_2}^{(8)}$ has a greater PRE in populations I, II, IV, VI, VIII, IX and X than the existing estimators, except for populations III, V and VII. From the Tables, it is observed that $t_{m1}^{(8)}$ has the greatest PRE of 2441.94%, 589.79%, 43924.52%, 231.97%, 717.24%, 190.71%, 709.54%, 129.13%, 339.07% and 930.69% among all existing and proposed estimators considered in this work. This is followed by $t_{m2}^{(8)}$ which is a member of the second proposed multivariate ratio estimator with 2342.28%, 576.57%, 231.94%, 283.2% and 924.89% in populations I, II, IV, IX and X respectively and $t_{m1}^{(7)}$ with PRE of 1405.31%, 710.29%, 190.22%, 705.70% and 128.08% in populations III, V, VI,VIII respectively . Therefore, $t_{m1}^{(8)}$ has the greatest gain in efficiency in all the ten populations.

Conclusion

Several classes of Asymptotic Optimal Estimators (AOE) of population mean, under the simple random sampling strategies, in the presence of two auxiliary variables, have been proposed in this work. However, $t_{m_1}^{(8)}$ and $t_{m_2}^{(8)}$ have demonstrated tremendous gains in efficiencies under this sampling strategies. They have therefore been found useful for estimating the population mean in simple random sampling strategy under certain optimal conditions.

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