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Comparative Study of Some Numerical Methods for Transcendental Equations

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Abstract

When analytical solutions are unavailable or difficult to find, numerical methods play an important role in solving linear and nonlinear problems. In this paper, we analyze and compare five numerical approaches typically utilized for solving linear and nonlinear problems arising in Physics, Engineering, Biosciences, etc. These are the Bisection method, Newton-Raphson method, Secant method, Regula-Falsi method, and Fixed-Point Iterative method. The weaknesses and strengths of each of these methods are examined and contrasted in terms of convergence, accuracy, computational efficiency, and applicability to different types of equations. The results of this research will assist academics and practitioners in making informed decisions on how to address algebraic and transcendental problems in their respective disciplines.

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Introduction

An expression of the form $f(x) = a_0x^n + a_1x^{n-1}+\ldots+a_{n-1}x + a_n$ where a's are constants ($a_0 \neq 0$) and n is a positive integer, is called a polynomial in x of degree n. The polynomial $f(x) = 0$ is called an algebraic equation of degree n. If $f(x)$ contains some other functions such as trigonometric, logarithmic, exponential etc., then $f(x) = 0$ is called a transcendental equation. The value α of x, which satisfies $f(x) = 0$ is called a root of $f(x) = 0$. Geometrically, a root of $f(x) = 0$ is that value of x where the graph of $y = f(x)$ crosses the x-axis. The process of finding the roots of an equation is known as the solution of that equation. This is a problem of fundamental importance in applied mathematics. If $f(x)$ is a quadratic, cubic, or biquadratic expression, algebraic solutions of equations are available. However, the need often arises to solve higher degree or transcendental equations without direct methods. Such equations can best be solved using approximate methods (Epperson, 2013; Otto and Deniar, 2005; Singh, 2011; Grewal, 2019 and Faires and Burden, 2013). These methods include the Bisection method, Newton-Raphson method, Secant method, Regula-Falsi method, Muller's method, and Fixed-Point Iterative method. Different researchers have made several attempts to find the right methods of solving these problems. Srivastava *et al.* (2011) and Ehiwario *et al* .(2014) respectively analyzed the efficiency of Newton-Raphson, Bisection, and Secant methods in solving nonlinear problems. The comparison of the results obtained showed the secant method as the most efficient (Srivastava and Srivastava, 2011; Ehiwario and Aghamie, 2014). In contrast, some other researchers settled on Newton-Raphson as the most efficient (Adegoke *et al*, 2018; Ebelechukwu *et al*., 2018; Moheuddin *et al*., 2019).
In this work, we seek to apply these existing numerical methods: the Bisection, Newton-Raphson, Secant, Regula-Falsi, and Fixed-Point Iterative methods, on four different problems drawn from polynomial, exponential, trigonometric and logarithmic functions. This will enable us to obtain an unbiased conclusion. The computations will be done manually and with the help of MATLAB software. This study aims to contribute to the understanding of the strengths and weaknesses of these different methods, and its findings will assist researchers and practitioners in making informed decisions regarding the selection of appropriate methods for solving algebraic and transcendental problems in their respective domains.

Numerical Methods

The Bisection Method*:*

Introduction: The bisection method is based on the continuity of the function f in a closed interval $[a, b]$. The repeated application of the intermediate theorem defines it. Supposed $f(a) \cdot f(b) < 0$, it means that a value α exists, between a and b in which f is zero. In other words, there exists a root α , (or more), of f

lying between a and b. Let the interval of the midpoint $[a, b]$ be given as:

$$c = \frac{a+b}{2} \tag{1}$$

and considering the product $f(a) \cdot f(c)$, the following possibilities exist:
$f(a) \cdot f(c) < 0$; this means $\alpha \epsilon [a, c]$.
$f(a) \cdot f(c) = 0$; since $f(a) \neq 0$, it means $f(c) = 0$.
$f(a) \cdot f(c) > 0$; this means $\alpha \epsilon [c, b]$.
If the second possibility (i.e. (ii)) is the case, which rarely happens, it means the root has been found. But if either (i) or (iii) is the case, the root is localized to an interval. $[a, c]$ or $[c, b]$ which is half the range of the original interval $[a, b]$. As such, the process will be repeated, and consequently, the uncertainty interval is decreased to half at each instance. This process is repeated until the root is localized within any desirable tolerance.

2.1.2 Properties of Bisection method: The bisection method has the following properties:
(i) It always converges no matter the interval.
(ii) Its rate of convergence is slow.
(iii) It is straightforward.

2.1.3 Convergence of Bisection Method: The Bisection method converges if:

$$|x_n - \delta| \leq K \frac{1}{2^n} \tag{2}$$

Where $K = |b - a|$ and $\frac{1}{2^n}$ is the convergence rate.

2.2 Newton-Raphson Method

2.2.1 Introduction: Newton-Raphson Method can be derived graphically or analytically (using Taylor's theorem). Graphically, the method improves on a given 'initial guess' to obtain a better-approximated root α for the function $y = f(x)$. It uses the tangent approximation to the function f at the point $(x_0, f(x_0))$.
From the point-slop equation of a straight line:

$$\frac{y-y_0}{x-x_0} = f'(x_0) \tag{3}$$

we have $y - y_0 = (x - x_0) \cdot f'(x_0)$
which gives $y = y_0 + (x - x_0) \cdot f'(x_0)$
or $y = f(x_0) + f'(x_0) \cdot (x - x_0)$
To solve for x_1, we let $y = 0$, then $f(x_0) + f'(x_0) \cdot (x - x_0) = 0$
Therefore, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
If this new approximate value is given, the value x_1, then repeating the process gives:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Thus, the general Newton-Raphson formula is given as:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{4}$$

Analytically, let $f(x_0) = 0$, and the approximate value of the required root be x_0, and h be the small increment to x_0, such that

$$x_1 = x_0 + h \tag{5}$$

Then $f(x_1) = 0$ implies $f(x_0 + h) = 0$. Applying Taylor's theorem on $f(x_0 + h) = 0$ yields:

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!} f'(x_0) + \ldots = 0$$

Neglecting h^2 and higher order of h, gives:

$$f(x_0) = f(x_0) + hf'(x_0) = 0$$

where

$$h = -\frac{f(x_0)}{f'(x_0)} \tag{6}$$

Substituting (6) in (5) gives:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{7}$$

as the first approximation to the required root.
Similarly,

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$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Thus, the general form of Newton-Raphson method is,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (8)$$

2.2.2 Properties of Newton-Raphson method: Newton-Raphson method has the following properties:

(i) It cannot be applied to a function f if $f'(x) = 0$.

(ii) Newton-Raphson's method converges fast when a right-guessed value is used, otherwise, the convergence rate will be slow, or convergence might not even exist.

(iii) If x_0 is complex, the method can be utilized to obtain a root.

2.2.3 Convergence of Newton-Raphson method: From the Newton-Raphson method, which is given as:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let

$$\emptyset(x) = x - \frac{f(x)}{f'(x)} = \frac{xf'(x) - f(x)}{f'(x)} \quad (8a)$$

Differentiate both sides of (8a) w.r.t. x:

$$\emptyset'(x) = \frac{f'(x)[f'(x) + xf''(x) - f'(x)] - [xf'(x) - f(x)]f''(x)}{[f''(x)]^2}$$

$$= \frac{[f'(x)]^2 + xf'(x)f''(x) - [f(x)]^2 - xf'(x)f''(x) + f(x)f''(x)}{[f''(x)]^2}$$

Therefore,

$$\emptyset'(x) = \frac{f(x)f''(x)}{[f''(x)]^2}$$

Since Newton-Raphson method is an iteration method, and the iteration method is said to converge if $|\emptyset'(x)| < 1$. Therefore, Newton-Raphson method converges if $|f(x)f''(x)| < [f'(x)]^2$, (Dass and Verma, 2012).

2.3 Secant Method

2.3.1 Introduction: The Secant method makes use of a secant line which passes through two points x_0 and x_1 with coordinates at $(x_0, f(x_0))$ and $(x_1, f(x_1))$. The two points denote two initial guesses whose roots will be used to approximate the next iterate, x_2. The equation of the line is given as:

$$\frac{y - f(x_1)}{x - x_1} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \quad (9)$$

If we let $y = 0$ and $x = x_2$, then solving for x_2:

$$(x_2 - x_1)\big(f(x_1) - f(x_0)\big) = (x_1 - x_0)\big(-f(x_1)\big)$$

$$x_2 = x_1 - f(x_1)\left[\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right]$$

In general, the Secant method is given as:

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right] \quad (10)$$

2.3.2 Properties of Secant method: Secant method does not require the function's derivative.

2.3.3 Convergence of Secant Method: From equation (10), we have the secant method as:

$$x_{n+1} = x_n - f(x_n)\left[\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right]$$

Let the approximate root of the function $f(x) = 0$ be δ. Then, substituting $x_n = \delta + \epsilon_n$ into the above equation gives:

$$\epsilon_{n+1} = \epsilon_n - f(\delta + \epsilon_n)\left[\frac{\epsilon_n - \epsilon_{n-1}}{f(\delta + \epsilon_n) - f(\delta + \epsilon_{n-1})}\right]$$

Expanding $f(\delta + \epsilon_n)$ and $f(\delta + \epsilon_{n-1})$ about δ using Taylor's series, and noting that $f(\delta) = 0$ gives: $\epsilon_{n+1} = \epsilon_n - \frac{(\epsilon_n - \epsilon_{n-1})\left[\epsilon_n f'(\delta) + \frac{1}{2}\epsilon_n^2 f''(\delta) + \cdots\right]}{(\epsilon_n - \epsilon_{n-1})f'(\delta) + \frac{1}{2}(\epsilon_n^2 - \epsilon_{n-1}^2)f''(\delta) + \cdots}$

That is:

$$\epsilon_{n+1} = \epsilon_n - \frac{\left[\epsilon_n f'(\delta) + \frac{1}{2}\epsilon_n^2 f''(\delta) + \cdots\right]}{\left[1 + \frac{1}{2}(\epsilon_{n-1} + \epsilon_n)\frac{f''(\delta)}{f'(\delta)} + \cdots\right]}$$

We have:

$$\epsilon_{n+1} = \frac{\frac{1}{2}\epsilon_n\epsilon_{n-1}f''(\delta)}{f'(\delta)} + 0(\epsilon_n^2\epsilon_{n-1} + \epsilon_n^2\epsilon_{n-1})$$

Which gives:

$$\epsilon_{n+1} = k\epsilon_n\epsilon_{n-1} \tag{11}$$

where $k = \frac{1}{2}\frac{f''(\delta)}{f'(\delta)}$ and neglecting the higher powers of ϵ_n (Moheuddin, 2019).

2.4 Regula-Falsi Method:

2.4.1 Introduction: The Regula-Falsi method is a hybrid Bisection-Secant approach. It entails selecting an initial interval [a, b] such that $f(a).f(b) < 0$. Then, join the two points with a chord. The equation of the chord at $(a, f(a))$ and $(b, f(b))$ is given as:

$$\frac{y-f(a)}{x-a} - \frac{f(b)-f(a)}{b-a} \tag{12}$$

Replacing part of the curve between the points $(a, f(a))$ and $(b, f(b))$ by the chord joining these points and taking the point of intersection of the curve with the axis gives an approximation to the root. Let $y = 0$ and $x = x_1$ in (10), we have:

$$(x_1 - a)(f(b) - f(a)) = -f(a)(b - a)$$

$$x_1 = a - f(a)\frac{(b-a)}{f(b) - f(a)}$$

By simplification, we have:

$$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} \tag{13}$$

as the first approximation of the root.

If $f(x_1).f(a) < 0$, then the root lies in the interval $[a, x_1]$. Thus, x_1 replaces b in (13), and the process is continued to obtain x_2 . But if $f(x_1).f(a) > 0$, it means the root lies in the interval $[x_1, b]$. Thus, x_1 replaces *a* in (13), and the equation is solved to obtain x_2. This procedure is repeated until the root is obtained to the desired accuracy.

2.4.2 Properties of Regula-Falsi Method: The Regula-Falsi method is slow. However, convergence is guaranteed.

2.4.3 Convergence of Regula-Falsi Method: Given an interval (a_n, b_n) such that the function $f(x)$ in the equation $f(x) = 0$ has its root, then one of the points a_n or b_n is always fixed and the other point varies with n, (Kumar, 2015). In the case where an is fixed, then the function $a_n, f(x)$ is approximated by a straight line which will pass through the points $(a_n, f(x_n))$ and$(x_n, f(x_n))$, for $n = 1,2, ...$

From the error equation

$$\epsilon_{n+1} = k\epsilon_n\epsilon_{n-1}$$

where $k = \frac{|f\prime\prime(\delta)|}{2|f\prime(\delta)|}$

And $\epsilon_0 = a_0 - \delta$ is independent of k. Hence, we can write

$$\epsilon_{n+1} = k'\epsilon_n$$

Where $k' = k\epsilon_0$ is the asymptotic error constant. This therefore implies that the Regula-Falsi Method has a linear convergence rate (Abdul-Hassan, 2016).

2.5 Fixed-Point Iterative Method

2.5.1 Introduction: in the iteration method, only one initial guess value of x is required. The

method can only be used if the function $f(x) = 0$ can be expressed as:

$$x = g(x) \tag{14}$$

Let the approximation to the desired root be x_0, then substituting this in the right-hand side of (14) and solving gives the first approximation as:

$$x_1 = g(x_0).$$

Repeating the procedure generates successive approximations as:

$$x_2 = g(x_1)$$
$$x_3 = g(x_2)$$

Generally, the iteration method is given as:

$$x_n = g(x_{n-1})$$

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2.5.2 Properties of Fixed-Point Iterative Method: The approximations sequence given by (14) can only converge to the root x in an interval I if

$$|g'(x)| < 1 \; for \; all \; x \in I$$

2.5.3 Convergence of Fixed-Point Iterative Method: Let g$\in C[a, b]$, such that $g(x) \in [a, b]$, for all $x \in [a, b]$. Assume g' exists on (a, b) and that a constant $0 \leq K \leq 1$ exists with $|g'(x)| \leq K$ for every $x \in (a, b)$. Then, for any number x_0 in $[a, b]$, the sequence defined by

$$x_n = g(x_{n-1})$$

converges to the unique fixed-point x in $[a, b]$, (Azurel et al 2019).

3. Application

In this section, we analyze the numerical methods discussed in this work by applying each method to each of the following examples:

i. $f(x) = x^2 + x - 4$ (15)

ii. $f(x) = x - e^{-x}$ (16)

iii. $f(x) = Cos\, x - 3x + 5$ (17)

iv. $f(x) = 2x - \log x - 7$ (18)

First, we will test for the possible root of each of the equations.

Example (i): $f(x) = x^2 + x - 4$

$f(0) = negative$; $f(1) = negative$

$f(1.5) = negative$; $f(1.6) = positive$

Since $f(1.5)f(1.6) < 0$, the root lies in the interval [1.5,1.6]. these initial guess values will be adopted for all the methods. The approximate root will be given to the accuracy of 0.0001. The results of the computations are presented on Table 1.

Example ii: $f(x) = x - e^{-x}$

$f(0.5) = negative$; $f(0.6) = positive$

Therefore, the root lies in the interval [0.5,0.6]. These initial values will be used for all the methods. The results obtained are presented on Table 2.

Example iii: $f(x) = Cos\, x - 3x + 5$

$f(0) = positive$; $f(1) = positive$

$f(2) = negative$; $f(1.5) = positive$

$f(1.6) = positive$; $f(1.7) = negative$

Therefore, the root lies in the interval [1.6, 1.7]. these initial guess values will be used for all the methods. The results are shown on Table 3.

Example iv: $f(x) = 2x - \log x - 7$

$f(3.5) * f(3.8) < 0$. Therefore, the root lies between 3.5 and 3.8. these guess values will be used for all the methods. The results are shown on Table 4.

Result and Discussion

Table 1: Root of the equation $f(x) = x^2 + x - 4$

n	Bisection	Newton-Raphson	Secant	Regula-Falsi	Fixed Point Iteration
1	1.550000	1.562500	1.560976	1.560976	1.581139
2	1.575000	1.561553	1.561547	1.561547	1.555254
3	1.562500	1.561553	1.561553	1.561553	1.563568
4	1.556250	1.561553	1.561553	1.561553	1.560907
5	1.559375	1.561553	1.561553	1.561553	1.561760
6	1.560938	1.561553	1.561553	1.561553	1.561486
7	1.561719	1.561553	1.561553	1.561553	1.561574
8	1.561328	1.561553	1.561553	1.561553	1.561574
9	1.561523	1.561553	1.561553	1.561553	1.561574
10	1.561621	1.561553	1.561553	1.561553	1.561574

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Table 1 gives the results obtained from the computation of (15). The Bisection method converged to 1.561621 with an accuracy of 0.0001 after the 10th iteration. The Newton-Raphson, Secant and Regula-Falsi methods converged to the value 1.561553 with an accuracy of less than 0.0001 after the 3rd iteration, respectively. At the same time, the Fixed-Point Iteration method converged after the 7th iteration to the value 1.561574 with accuracy equal to 0.0001.

Table 2: Root of the equation $f(x) = x - e^{-x}$

n	Bisection	Newton-Raphson	Secant	Regula-Falsi	Fixed Point Iteration
1	0.550000	0.566311	0.600000	0.567545	0.606531
2	0.575000	0.567143	0.567549	0.567165	0.545239
3	0.562500	0.567143	0.567141	0.567143	0.579703
4	0.568750	0.567143	0.567144	0.567143	0.560065
5	0.565625	0.567143	0.567144	0.567143	0.571172
6	0.567188	0.567143	0.567144	0.567143	0.564863
7	0.566407	0.567143	0.567144	0.567143	0.568438
8	0.566798	0.567143	0.567144	0.567143	0.566409
9	0.566600	0.567143	0.567144	0.567143	0.567560
10	0.566600	0.567143	0.567144	0.567143	0.566907
11	0.566600	0.567143	0.567144	0.567143	0.567277
12	0.566600	0.567143	0.567144	0.567143	0.567067
13	0.566600	0.567143	0.567144	0.567143	0.567187

The results obtained from the computation of (16) is presented in Table 2. The Bisection method converged to 0.566600 with an accuracy of 0.0001 after the 9th iteration. The Newton-Raphson and Regula-Falsi methods converged to the value 0.567143 with accuracy less than 0.0001, respectively, after the 3rd iteration. The Secant method converged to 0.567144 after the 4th iteration with an accuracy of less than 0.0001. Then, the Fixed-Point Iteration method converged after the 13th iteration to the value 0.567187 with accuracy equal to 0.0001.

Table 3: Root of the equation $f(x) = Cos\ x - 3x + 5$

n	Bisection	Newton-Raphson	Secant	Regula-Falsi	Fixed Point Iteration
1	1.650000	1.642705	1.642738	1.642738	1.656933
2	1.625000	1.642715	1.642715	1.642715	1.637990
3	1.637500	1.642715	1.642715	1.642715	1.644286
4	1.643750	1.642715	1.642715	1.642715	1.642192
5	1.640625	1.642715	1.642715	1.642715	1.642888
6	1.642187	1.642715	1.642715	1.642715	1.642657
7	1.642969	1.642715	1.642715	1.642715	1.642734
8	1.642578	1.642715	1.642715	1.642715	1.642734
9	1.642773	1.642715	1.642715	1.642715	1.642734
10	1.642676	1.642715	1.642715	1.642715	1.642734

Table 3 shows the results obtained from the computation of (17). The Bisection method converged to 1.642676 with an accuracy of 0.0001 after the 10th iteration. The Newton-Raphson, Secant and Regula-Falsi methods converged to the value 1.642715 each with accuracy less than 0.0001 after the 3rd iteration, respectively. In contrast, the Fixed-Point Iteration method converged after the 7th iteration to the value 1.642734 with an accuracy equal to 0.0001.

Table 4: Root of the equation $f(x) = 2x - log x - 7$

n	Bisection	Newton-Raphson	Secant	Regula-Falsi	Fixed Point Iteration
1	3.650000	3.500000	3.800000	3.800000	3.772034
2	3.725000	3.817373	3.789252	3.734617	3.788288
3	3.762500	3.786755	3.789279	3.789274	3.789221
4	3.781250	3.789495	3.789279	3.789278	3.789275
5	3.790625	3.789259	3.789279	3.789278	3.789275
6	3.785936	3.789280	3.789279	3.789278	3.789275
7	3.788281	3.789280	3.789279	3.789278	3.789275
8	3.789453	3.789280	3.789279	3.789278	3.789275
9	3.788867	3.789280	3.789279	3.789278	3.789275
10	3.789160	3.789280	3.789279	3.789278	3.789275
11	3.789306	3.789280	3.789279	3.789278	3.789275
12	3.789233	3.789280	3.789279	3.789278	3.789275

The results obtained from the computation of (18) is presented in Table 4. The Bisection method converged to 3.789233 with an accuracy of 0.0001 after the 12th iteration. Newton-Raphson, Secant, Regula-Falsi, and Fixed-Point Iteration methods converged to 3.789280, 3.789279, 3.789278, and 3.789275 after the 6th, 3rd, and 4th iterations, respectively, with accuracy less than 0.0001.

Conclusion

Looking closely at the results presented above (Tables 1-4), which were obtained from the computations of different nonlinear equations drawn from polynomial and transcendental equations, we observed that the rate of convergence with Newton-Raphson, Secant, and Regula-Falsi methods is almost the same; followed by Fixed Point Iteration method and the Bisection method. These findings contrdict the findings of Azure et al (2019) who placed Newton-Raphson and Regula-Falsi methods as more efficient than the Secant method. On the contrary, some other authors placed the Secant method ahead of the Newton-Raphson and Regula-Falsi methods (Srivastava and Srivastava, 2011) In summary, we can conclude that Newton-Raphson, Secant, and Regula-Falsi methods have proven to be the most efficient, no matter the nature of the nonlinear problem under consideration.

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