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Dufour and Soret Effects on the Onset of Thermosolutal Instability in a Rotating Porous Layer

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Abstract

The Study investigates the Dufour and Soret effect on the onset of thermosolutal instability in a rotating porous layer using Darcy's law and Linear Stability analysis. When the rotation Lewis number, Solutal Rayleigh number Dufour parameter, and Soret parameter, are set equal to zero, It was observed that the principle of exchange of stabilities holds and hence found that instability manifested itself as stationary convection. Rotation and Dufour parameter respectively have a destabilizing effect on the system as the Rayleigh number, increases with an increase in Rotation and Dufour parameters for all values of the wave number and Solutal Rayleigh number while the Soret parameter has a stabilizing effect on the system as the Rayleigh number decreases with increase in Soret parameter

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Introduction

According to (Eckert and Drake, 1972) mass flux created by a temperature gradient is known as the soret or thermal–diffusion effect whereas the energy flux caused by a composition gradient is called the Dufour or diffusion–thermo effect. The thermal–diffusion (soret) effect, for instance, has been utilized for isotope separation, and in a mixture of gases with very high molecular weight (H_2, He) and of medium molecular weight (N_2, air). The diffusion–thermo (Dufour) effect was found to be of a considerable magnitude such that it cannot be ignored (Eckert and Drake, 1972).

Postelnicu (2004) has considered both Soret and Dufour effects on the natural convection about a vertical surface embedded in a saturated medium subjected to a magnetic field. Vaidyanathan *et al.*(1997) studied Ferro thermohaline convection in which a horizontal layer of an incompressible ferromagnetic fluid of thickness d, heated from below and salted from above in the presence of a transverse magnetic field was considered. Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of a uniform vertical magnetic field has been considered by Finlayson (1970). He explained the concept of thermo-mechanical interaction ferromagnetic fluids. From the wide range of applications of ferromagnetic fluid to instrumentation, lubrication, printing, vacuum technology, vibration damping, metals recovery, acoustics and medicine. Hill (2005) investigated the problem of double-diffusive convection in a porous medium with a concentration-based internal heat source using linear and nonlinear stability analyses. He employed Darcy's Law and Boussinesq's approximation with the equation of state taken to be linear to temperature and concentration.

Kaufman (1994) focused on convective instabilities of through flow in packed beds with internal heat sources where a packed bed is heated with internal heat sources. Bdzil and Frisch (1971) discussed the effect of suction on heat transfer with internal generation or absorption in a viscous flow at a stagnation point.

Sharma and Rana (2002) considered the thermosolutal instability of Walters' (model B') elastico-fluid in a porous medium in the presence of uniform rotation, suspended particles and variable gravity field. There has been some work conducted on the exploration of this system namely Krishnamurti (1997) and Straughan (2002) for a viscous fluid and Hill (2003) for a fluid-saturated porous medium. Each of these bodies of work defines the state equation to be linear in temperature, with the effect of concentration on density assumed to be negligible. Hill (2005) explored the use of a double-diffusive convection model in a porous, with fixed boundary conditions employed. In addition to establishing the double-diffusive convection model, he developed a complementary energy theory. The energy method has been employed much success in double-diffusive flows, with examples including Carr (2003), Guo and Kaloni (1995) and Mulone and Rionero (1997). Bahadori and Rashidi (2012), investigated analytically thermo-diffusion and diffusion-thermo effects on heat and mass transfer in a two-layered cavity.

Taslim and Nasurawa (1986) and Malashetty (1993) used linear stability analysis to investigate the onset of convection in a double-diffusive flow. The work was later extended by Nield and Beijan (2006) to consider the effects of inclined temperature and solutal gradients. It was observed that both the thermal and solutal Rayleigh numbers contributed significantly to the onset of convective instability. Shivaiah and Rao (2011) analysed the effects of Soret Dufour and thermal radiation on unsteady magneto-hydrodynamic free convection flow past an infinite vertical porous plate in the presence of a chemical reaction. In all these investigations and studies, the Dufour and Soret effects on the onset of thermosolutal instability in a rotating porous layer has not been treated and constitute an important addition to the area of porous media convection studies.

Mathematical Formulation

The lower and upper surfaces are maintained at temperatures $T' \; and \; T_0$ and soluted mass concentrations $C' \; and \; C_0$ respectively. The whole system is assumed to be rotating with angular velocity $\Omega = (0,0,\Omega)$ along the vertical axis which is taken as the z-axis. The fluid is assumed to be Newtonian and the porous medium obeys Boussinesq's approximation and also the flow is governed by Darcy's law. Further, it is assumed that the fluid density ρ, is linearly dependent on the temperature T and concentration C according to Lawson and Yang (1973), and Hill (2005).Hence the density equation of state is taken as:

$$\rho\,(T'\,C') = \rho_0\,[1-\beta_T\,(T'-T_0) + \beta_c\,(C'-C_0)] \qquad (1)$$

where, ρ_0 is the reference density of the rotating fluid at temperatures $T' = T_0$ and mass fractions $C' = C_0$, T_0 and C_0 are reference temperature and concentration respectively, $\beta_T \; and \; \beta_c$ are thermal expansivities for temperature and solute concentrations respectively. We consider the porous layer to be arbitrarily close to the axis of rotation and hence we assume that the gravity buoyancy is dominant and centrifugal buoyancy is negligible.

Mathematical Equations

The mathematical equations governing the motion of the fluid-saturated by the porous medium, following Boussinesq's approximation for the above model according to Chandrasekhar (1981) and Nield and Bejan (2006) are:

Continuity Equation

$$\vec{\nabla}'.\vec{v}' = 0 \qquad (2)$$

Momentum Equation

$$\vec{\nabla}'P' + \frac{\mu}{k}\,\vec{v}' + \rho\,(T',C')g\vec{k} + 2\rho_o\,\hat{\Omega}\,k\times\vec{v} = 0 \qquad (3)$$

Energy Equation

$$(\rho_0 C_p)_m\,\frac{\partial T'}{\partial t'} + (\rho_0 C_p)_f\,\vec{v}'.\vec{\nabla}'T' = k_m\,\vec{\nabla}'^2 T' + D_{TC}\,\vec{\nabla}'^2\,c' \qquad (4)$$

Concentration Transport Equation

$$\Phi\,\frac{\partial c'}{\partial t'} + \vec{v}'\vec{\nabla}'C' = D_m\,\vec{\nabla}'^2\,C' + D_{CT}\vec{\nabla}'^2 T' \qquad (5)$$

Using equation (1) into (2) - (4), the governing equations become

$$\vec{\nabla}'.\vec{v}' = 0 \qquad (6)$$

$$\vec{\nabla}'\left(P' + \rho_o\,gz'\right) + \frac{\mu}{K}\vec{v}' - \rho o\,g\beta_T\,(T'-T_0)\,\hat{k}$$

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$$+ \ (\rho_o \, g\beta_c \, (c' - c_0)\hat{k} + \ 2\rho_0 \vec{\Omega} \, \hat{k} \times \vec{v}' \ = \ 0 \tag{7}$$

$$A\frac{\partial T'}{\partial t'} \ + \vec{v}\vec{\nabla}' T' = \ \frac{k_m}{(\rho C_P)_f} \ \vec{\nabla}'^2 T' \ + \ \frac{D_{TC}}{(\rho Cp)_f} \vec{\nabla}'^2 C' \tag{8}$$

$$\Phi \frac{\partial c'}{\partial t'} \ + \ \vec{v}' . \vec{\nabla}' C' \ = \ D_m \ \vec{\nabla}'^2 C' \ + D_{CT} \vec{\nabla}'^2 T' \tag{9}$$

where, $A = \dfrac{(\rho_0 C_P)_m}{(\rho_0 C_P)_f}$

We solve equations (6 – 9) subject to the following boundary conditions:

$$w' = \ 0, \quad T' = T_{1,} \quad C' = C_1 \quad \text{at } z = -\frac{h}{2}$$

$$w' = 0 \ , \quad T' = T_{2,} \quad C' = C_2 \quad \text{at } z = + \ \frac{h}{2} \tag{10}$$

2.2. Non-dimensionalization

Equations (6 –9) together with equation (10) are written in dimensionless form as

$$\nabla . \vec{v} = 0 \tag{11}$$

$$\nabla p + \vec{v} - R_T T + R_C C + T_a \hat{k} \times \vec{v} \ = 0 \tag{12}$$

$$\frac{\partial T}{\partial t} \ + \ \vec{v} \, . \nabla T \ = \ \vec{\nabla}^2 T \ + D_f \nabla^2 C \tag{13}$$

$$b\frac{\partial c}{\partial t} \ + \ \vec{v} . \nabla C \ = \ \frac{1}{Le} \vec{\nabla}^2 \ + \ S_r \vec{\nabla}^2 T \tag{14}$$

Subject to the boundary condition

$$w = 0 \, , \quad T = \pm\frac{1}{2}, \ c = \pm\frac{1}{2} \ \text{ on } z = \pm\frac{1}{2} \tag{15}$$

In equation (11) – (14) and the boundary condition (15) the following dimensionless variables were introduced.

$$(x, y, z,) = \frac{1}{h}(x', y', z'), \qquad t = \frac{\alpha_m t'}{Ah^2}, \qquad \vec{v} = \frac{h\vec{v}'}{\alpha_m}$$

$$\alpha_m = \frac{k_m}{(\rho c_p)_f} \ , \qquad \nabla' = \frac{1}{h}\nabla, \qquad p = \frac{(p' + \rho_0 g z')k}{\mu \alpha_m}$$

$$R_T = \frac{\rho_0 g \ \beta_T \ k \ h \, (T_1 - T_2)}{\mu \alpha_m} \quad = \text{Thermal Rayleigh number}$$

$$R_C = \frac{\rho_0 g \ \beta_c \ kh \, (C_1 - C_2)}{\mu \alpha_m} \quad = \text{Solutal Rayleigh number}$$

$$T_a = \frac{2 \, \hat{\Omega} \, \rho_0 \ k}{\mu} \ , \qquad T = \frac{T' - T_0}{T_1 - T_2}, \quad C = \frac{C' - C_0}{C_1 - C_2}$$

$$D_f = \frac{D_{TC}(C_1 - C_2)}{\alpha_m \ (\rho C_p)_f (T_1 - T_2)} \quad = \text{Dufour parameter}$$

b = $\frac{\Phi}{A}$, $S_r = \frac{D_{CT}(T_1 - T_2)}{\alpha_m (C_1 - C_2)}$ = Soret number

$\frac{\alpha_m}{D_m}$ = Le = Lewis number

Method of Analysis

Steady State

We seek an initial steady state solutions for which $\vec{V} = 0$ and $\frac{\partial}{\partial t} \rightarrow 0.$

Putting into equations (11) to (14) and the boundary conditions (15) and setting $T = T_B, \quad C = C_{B,}$ where the subscripts represent basic state, we obtain

$$\frac{dP_B}{dz} = R_T T_B - R_c C_B \tag{16}$$

$$\frac{d^2T_B}{dz^2} + D_f \frac{d^2C_B}{dz^2} = 0 \tag{17}$$

$$\frac{d^2C_B}{dz^2} + Le\ Sr \frac{d^2T_B}{dz^2} = 0 \tag{18}$$

Subject to the conditions

$$T_B = \pm\frac{1}{2}, \quad C_B = \pm\frac{1}{2} \quad on \quad z = \mp \frac{1}{2}$$

Putting equation (18) into (17), we have

$$\frac{d^2T_B}{dz^2} + D_f \left(- Le\ Sr \frac{d^2T_B}{dz^2} \right) = 0 \tag{19}$$

where C_1 and C_2 are constants of integration. Applying condition (19),

$$T_B = -z \tag{20}$$

Substituting (20) into (18), we have

$$\frac{d^2c_B}{dz^2} = 0$$

From this we obtain

$$C_B = -z \tag{21}$$

Therefore,

$$p_B = \int (-R_T z + R_C z) dz,$$

Hence, $p_B = \frac{(R_C - R_T)z^2}{2}$ (22)

Linearization and Perturbation Equation

To study the stability of the steady-state solution, we assume a small disturbance around the basic solution (Chandrasekhar, 1981, Drazin and Reid, 2004);

$$\vec{V} = 0 + \vec{V}$$

$$T = T_B + \theta$$

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$$C = C_B + \varphi$$

$$P = P_B + p \tag{23}$$

where $\theta << T_{B,}$, φ << C_B, $P << \rho_B$,

Substituting (3.23) into equation (3.11), (3.14) and the boundary conditions (15), we obtain

$$\vec{\nabla} . \vec{v} = 0 \tag{24}$$

$$\nabla P = (R_T\theta - R_C\varphi)\vec{k} - \vec{v} - T_a\hat{k}\times\vec{v} \tag{25}$$

$$\frac{\partial\theta}{\partial t} = \vec{\nabla}^2\theta + D_f\vec{\nabla}^2\varphi + w \tag{26}$$

$$b\frac{\partial\varphi}{\partial t} = \frac{1}{Le}\left(\vec{\nabla}^2\varphi + S_r\vec{\nabla}^2\theta\right) + w \tag{27}$$

Subject to the boundary condition

$$W = \theta = \varphi = 0 \quad \text{on,} \quad z = \pm\frac{1}{2} \tag{28}$$

Next, eliminating the pressure perturbation by operating double curl on (25) using the continuity equation (24), and taking only the z-component yields

$$T_1\frac{\partial w}{\partial z} = -\xi \tag{29}$$

$$\nabla^2\omega = R_T\nabla_h^2\theta - R_C\nabla_h^2\varphi - T_a\frac{\partial\xi}{\partial z} \tag{30}$$

where

$\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, is the Laplacian in the horizontal plane

$\xi = \frac{\partial v}{\partial x^2} - \frac{\partial u}{\partial y}$, is the z – component of the vorticity

Normal Mode Analysis

Next, we examine the stability of small perturbations from the inactive state of the situation described by (12)–(15). The analysis is made in terms of two–dimensional periodic waves of assigned wave numbers. Here, we assume the perturbed solution to have a wavelike disturbance of the form

$$w = W(z)e^{ht+i(k_x^x+k_y^y)}$$

$$\theta = \Theta(z)e^{ht+i(k_x^x+k_y^y)},$$

$$Z = z(z)e^{ht+i(k_x^x+k_y^y)}$$

$$\varphi = \Psi(z)e^{ht+i(k_x^x+k_y^y)} \tag{31}$$

where k_x and k_y are wave numbers in the x and y directions respectively and h is a constant, which can be complex,

$h = ih_i$,

Substituting equation (31) into (26), (27), (29) and (30), we obtain

$$T_a\frac{dw}{dz} = -z \tag{32}$$

$$(D^2 - k^2)w = R_T k^2 \theta - R_c k^2 \varphi - T_a Dz \quad (33)$$

$$(D^2 - k^2 - h)\Theta + D_f (D^2 - k^2)\varphi + w \quad (34)$$

$$(D^2 - k^2 - Lebh)\Psi + S_r (D^2 - k^2)\Theta + Lew = 0 \quad (35)$$

where, $\frac{\partial}{\partial t} = h, \quad \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = k^2, \quad D = \frac{d}{dz},$

$D^2 = -\frac{\partial^2}{\partial z^2}, \quad \nabla^2 = D^2 - k^2, \quad k^2 = k_x^2 + k_y^2$

Equation (32) can be written in the form

$$T_a Dw = -z \quad (36)$$

So that

$$-T_a D^2 w = +Dz \quad (37)$$

Substituting (3.37) into (3.33), we get

$$(D^2 - k^2)w = R_T k^2 \Theta - R_C k^2 \Psi - (T_a)^2 D^2 w \quad (38)$$

To proceed with the analysis we seek solutions of the form

$$(\Theta, \Psi,) = (\Theta_0, \Psi_0, w_O)\sin n\pi z, \quad (39)$$

Where n is a positive integer and where $\Theta_0, \Psi_0, and\ w_0$ are in general complex. Substituting equation (39) into (34), (35) and (38), yield:

$$(n^2\pi^2 + k^2 + \Omega)\Theta_0 + D_f (n^2\pi^2 + k^2)\varphi_0 - w = 0 \quad (40)$$

$$LeS_r \left(n^2\pi^2 + k_2\right)\Theta_0 + \left(n^2\pi^2 + k^2 + Leb\Omega\right)\varphi_0 - Lew_0 = 0 \quad (41)$$

$$R_T k^2 \Theta - R_C k^2 \varphi_0 + (n^2\pi^2 T_a^{\ 2} + n^2\pi^2 + k^2)w_0 = 0 \quad (42)$$

For an idealized fluid layer with free boundaries for which n=1, we can rewrite equations (40) - (42) in matrix form as

$$\begin{pmatrix} J+\Omega & D_f J & -1 \\ LeS_r J & J + Leb\Omega & -Le \\ R_T k^2 & -R_C k^2 & J + \pi^2 T_a^{\ 2} \end{pmatrix} \begin{pmatrix} \Theta \\ \varphi_0 \\ w_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (43)$$

where $J = \pi^2 + k^2$

The solvability of the system (43) requires that

$$\begin{vmatrix} J+\Omega & D_f J & -1 \\ LeS_r J & J + Leb\Omega & -Le \\ R_T k^2 & -R_C k^2 & J + \pi^2 T_a^{\ 2} \end{vmatrix} = 0$$

That is,

$$\left(J + bLe\Omega - JLeD_f\right)R_T k^2 + \left(JLeSr - JLe - Le\Omega\right)R_C k^2 + \left(J + \pi^2 T_a^{\ 2}\right)\left(J\Omega + J^2 + bLe\Omega^2 + JbLe\Omega - J^2 LeD_f S_r\right) = 0 \quad (44)$$

By rearranging equation (44), we obtain an expression for the characteristic value of the thermal Rayleigh number as:

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$$R_T = \frac{\left(J + \pi^2 T_a^{\ 2}\right)\left(D_f J^2 LeS_r - J^2 - JLeb\Omega - J\Omega - Leb\Omega^2\right)}{K_2\left(J - JLeD_f + Leb\Omega\right)} + R_C Le\left(\frac{J - JS_r + \Omega}{J - JLeD_F + Leb\Omega}\right) \tag{45}$$

Onset of Stationary and Oscillatory Convection

Having established the thermal Rayleigh number, we proceed to study the stationary and oscillatory convection.

Stationary convention

For stationary convention, the marginal state is characterized by $\Omega = 0$, and setting $R_T = R_T(S)$ in equation (45), the Rayleigh number becomes:

$$R_T(s) = J\left(\frac{J + \pi^2 T_a^{\ 2}}{k^2}\right)\left(\frac{1 - D_f LeS_r}{1 - D_f Le}\right) + R_C Le\left(\frac{1 - S_r}{1 - D_f Le}\right) \tag{46}$$

Next, we seek for the critical wave number, k_C, for the onset of stationary instability in the system by setting $k = k_C$ in equation (46) following Chandrasekhar (1961).

Simplifying gives:

$\frac{\partial R_T(s)}{\partial k^2} = 0$, which yields the critical wave number:

$$k_C^{\ 2} = \pi^2\sqrt{1 + Ta^2}, \qquad k^2 > 0 \tag{47}$$

substituting equation (47) into equation (46) gives the critical Rayleigh number for stationary convections

$$R_{T\,Cri}(S) = \left(2\sqrt{1 + T^2} + T_a^{\ 2} + 2\right)\left(\frac{D_f LeS_r - 1}{1 - D_f Le}\right)\pi^2 + R_C Le\left(\frac{1 - S_r}{1 - D_f Le}\right) \tag{48}$$

Oscillatory Convection

For marginal stability, set $\Omega_r = 0$ so that the oscillatory frequency contains only the imaginary part where $\Omega i = w$. Substituting $\Omega = iw$ into equation (45), we obtain the oscillatory thermal Rayleigh number. Substituting $\Omega = iw$, into equation (45), yields the oscillatory thermal Rayleigh number

$$R_T(0) = -\left[\frac{(\pi^2 + k^2 + \pi^2 T_a^2)\left\{(\pi^2 + k^2)^2(1 - D_f LeS_r) - Leb\omega^2 + i\omega(\pi^2 + k^2)(1 + Leb\right\}}{k^2\left(\pi^2 + k^2 + Lebiw - \left(\pi^2 + k^2\right)LeD_f\right)}\right.$$
$$\left. + R_2\left(\frac{\left(\pi^2 + k^2\right)LeS_r - \left(\pi^2 + k^2\right)Le - Leiw}{\pi^2 + k^2 + Lebiw - \left(\pi^2 + k^2\right)LeD_f}\right)\right] \tag{49}$$

Putting $J = \pi^2 + k^2$, equation (49) reduces into

$$R_T(0) = \left[\frac{\left(J + \pi^2 T_a^{\ 2}\right)\left(J^2 - D_f J^2 LeS_r + JLebiw + Jiw - Lebw^2\right)}{k^2\left(J - JLeD_f + Leiw\right)} + R_C\left(\frac{JLeS_r - JLe - Leiw}{J - JLeD_f + Leiw}\right)\right] \tag{50}$$

Simplifying equation (50) and separating the real and imaginary parts yields

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$$\mathrm{Re}\{R_T(0)\} = \frac{(1-De_f Le)J^2}{k^2(J-JLeD_f)^2-(Lebw)^2}\left[J(J+\pi^2Ta^2)(D_f LeS_r-1)+R_C Lek^2(1-S_r)\right]$$
$$-\frac{LebJ(J+\pi^2Ta^2)(D_f+b)w^2}{k^2(J-JLeD_f)^2-(Lebw)^2}+\frac{R_C Lebw^2}{(J-JLeD_f)^2-(Lebw)^2} \qquad (51)$$

$$\mathrm{Im}\{R_T(0)\} = \frac{R_C LeJ(1-D_f Le)(1-Leb)w}{(J-JLeD_f)^2-(Lebw)^2}$$
$$-\frac{(J+\pi^2Ta^2)}{k^2(JLeD_f)^2-(Lebw)^2}\left[J\{(Leb+1)(1-D_f Le)+(D_f LeS_r-1)Leb\}w\right] \qquad (52)$$

Equating equation (52) to zero and simplifying gives

$$w^2 = \frac{J(J+\pi^2Ta^2)\left[Leb\{2-D_f Le(S_r+1)\}-(D_f Le-1)\right]}{(Leb)^2}$$
$$-\frac{R_C LeJK^2(1-D_f Le)(1-Leb)}{(Leb)^2} \qquad (53)$$

Results and Discussion

This article showed an analytical solution to the formulated problem and numerical simulation done using Mathematica version 9. The simulations are shown below for various values of the parameters.

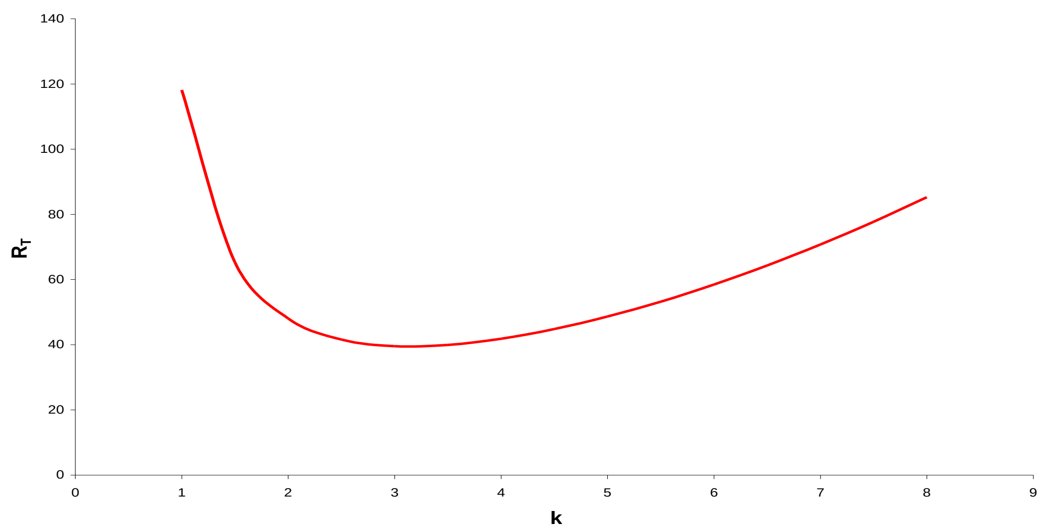

Figure 4.1 **: The Variation of Rayleigh numbers (R_T) with wave number (k)**

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https://doi.org/10.5555/IYGL7896 **ISSN 2488-8648**

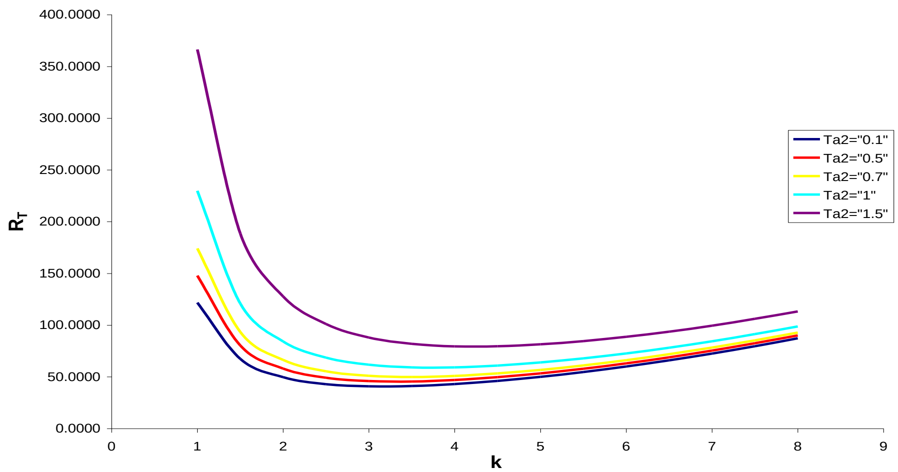

Figure 4.2 **: Values of Rayleigh number (R_T) and wave numbers (k) for various values of rotation T_a^2 for the onset of stationary convection**

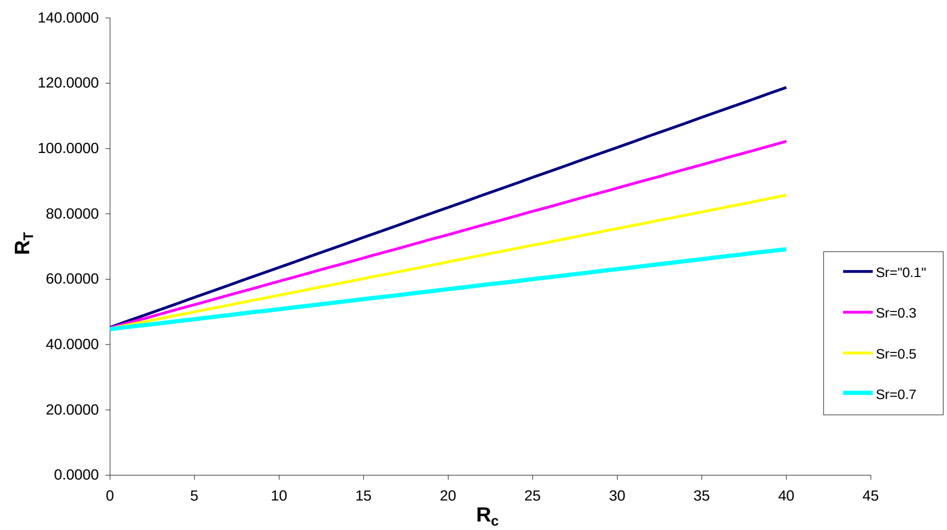

Figure 4.3 **: The Variation of Rayleigh number (R_T) with Solutal Rayleigh number R_C)**

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May, Volume 10, Number 2, Pages 159 - 170 http://www.ijbst.fuotuoke.edu.ng/ 168
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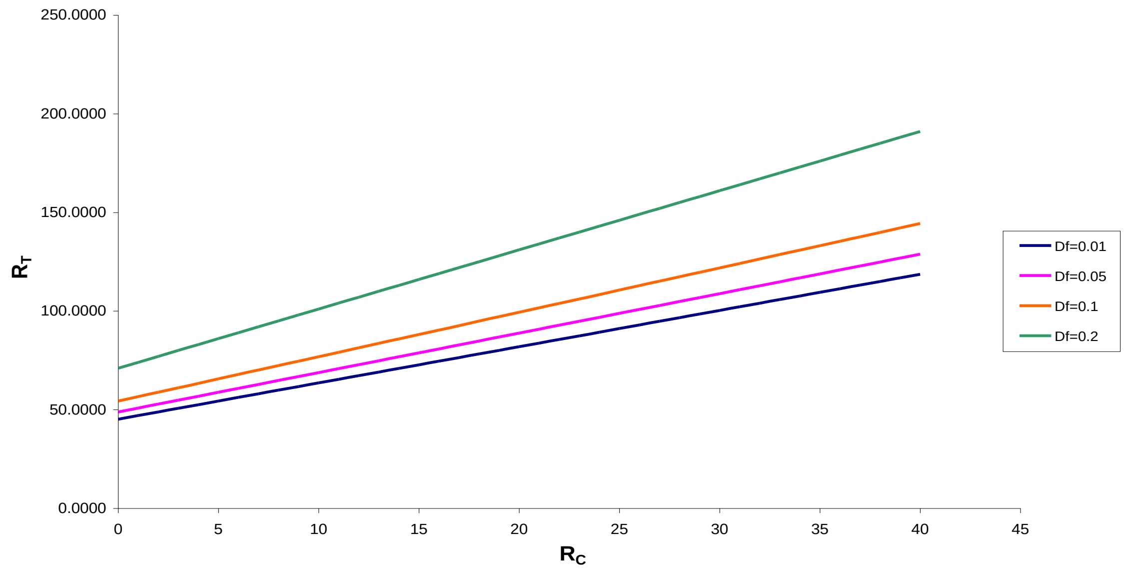

Figure 4.4 **: Values of Rayleigh number (R_T) for various values of Solutal Rayleigh number (R_C)**

Fig.4.1 represents the plot of Rayleigh number (R_T), for the onset of stationary convection versus the wave number (k). Considering equation (47), that is,

$$k_C^{\ 2} = \pi^2 \sqrt{1 + Ta^2}\,, \qquad k^2 > 0$$

In the absence of Rotation ($T_a^{\ 2}$), Lewis number (Le), Solutal Raleigh number (R_C), Dufour parameter (D_f) and Soret parameter (S_r), that is, $T_a^{\ 2} = Le = R_c = D_f = S_r = 0$, equation (47) becomes: $k_c = \pi$

$$R_T(s) = 4\pi^2$$

This is consistent with the result previously reported by Lambardo *et al.* (2003), Isreal – Cookey and Omubo Pepple (2011). When the Rayleigh number is less than or reaches a given value, the disturbances become stable and when they exceed the threshold the disturbance become unstable (Hill, 2005 and Lambardo *et al*. (2003)) .Fig.4.2 shows the variation of Rayleigh number (R_T) number with the wave number (k) for fixed values of Lewis number (Le), Solutal Rayleigh number (R_C), Dufour parameter (D_f) and Soret parameter (S_r) for various values of Rotation($T_a^{\ 2}$). Fig.4.3 shows variation of Rayleigh number (R_T) with Solutal Rayleigh number (R_C) for fixed values of Rotation ($T_a^{\ 2}$), Lewis number (Le), Dufour parameter (D_f) and wave number (k) for various values of Soret (S_r). Fig4.4 shows the variation of Rayleigh number (R_T) with Solutal Raleigh number (R_C) for fixed values of Rotation ($T_a^{\ 2}$), Lewis number (Le), Soret parameter (S_r) and wave number (k) and for various values of Dufour (D_f). The results show that increase in Rotation ($T_a^{\ 2}$) and Dufour (D_f) can cause an increment in the Rayleigh number for stationary convection. This means that Rotation ($T_a^{\ 2}$) and Dufour (D_f) delay the onset of instability in the system while an increment in Soret causes a decrease in the Rayleigh number for stationary convection. This means that an increase in Soret parameter hastens the onset of instability in

May, Volume 10, Number 2, Pages 159 - 170 http://www.ijbst.fuotuoke.edu.ng/ 169
https://doi.org/10.5555/IYGL7896 ISSN 2488-8648

the system. Lambardo *et al*. (2003) and Isreal-Cookey (2006).

Conclusion

The study was carried out to investigate the Dufour and Soret effects on the onset of thermosolutal instability in a rotating porous layer by representing the problem mathematically. Darcy's law and stability analysis were used for the investigation. The analytical solutions were analyzed and simulation was done using Wolfram Mathematica, focusing on varying the relevant parameters and the results were revealed as follows:

1. When the Rayleigh number was less than or equal to the threshold value, the disturbances became stable but when they exceeded the threshold the disturbance became unstable.
2. An increase in Rotation (T_a^2) and Dufour (D_f) can cause an increment in the Rayleigh number for stationary convection.
3. Rotation (T_a^2) and Dufour parameters (D_f) respectively has a destabilizing effect on the system as the Rayleigh number (R_T) increases with an increase in Rotation (T_a^2) and dufour parameters (D_f) for all values of the wave number (k) and Solutal Rayleigh number (R_C).
4. Increase in Soret parameter (S_r) hastens the onset of instability in the system.

Recommendations

Natural extensions of the stability analysis include considering other boundary conditions at the upper and lower surfaces or studying the role of the effects of varying values of Dufour, Soret, thermal Rayleigh number and inertial parameter. The examination of the effects of radiation on a porous layer, are also of topical interest and offer additional interesting possibilities for further study.

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