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A logical perspective to Soft Set Theory

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Abstract

The research discusses numerous mathematical solutions for coping with uncertainty, as well as their flaws. The concept of a Soft set, which is free of such flaws, was developed, along with some of its characteristics. It is shown that, unlike a classical set, a soft set may not contain all of the elements in its soft subset. Finally, the usage of soft sets in different theories was investigated, including proposition logic, fuzzy sets, topological spaces, and rough sets.

Key Words
soft set, fuzzy set, rough set, soft tautology, soft contradiction

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Introduction

Molodtsov (1999) introduced the theory of soft Set as a general mathematical tool for dealing with uncertain, Fuzzy ,and not clearly defined objects. Before the introduction of soft set theory, there were several existing theories for dealing with problems of uncertainty in mathematics. Some of those theories include the theory of fuzzy sets (Zadeh, 2005), the theory of intuitionistic fuzzy sets (Atanassov, (1986)), theory of vague sets (Gau and Buehrer, 1993), the theory of interval mathematics (Gorzalzany, 1987) , and theory of rough sets (Pawlak and Skowron, 2007). .However, these theories have associated difficulties or challenges arising from the inadequacy of parameterization tools in their applications. Molodtsov (1999) initiated the concept of soft set theory as a new mathematical alternative that is free from the inadequacy of the parametrization tools seen in the other mathematical tools for dealing with uncertainties. He further showed that the theory can be applied to several areas successfully; for example, the smoothness of functions, game theory, Riemann-integration, Perron-integration, amongst others.

A quick review of three prior methods used for the investigations of mathematical uncertainties and the challenges peculiar to them will be necessary to aid our understanding of the subsequent discussions.

Some Mathematical tools dealing with uncertainties

Theory of Probability

Probability theory as in Molodtsov (1999) is concerned with the analysis of random phenomena; this random phenomenon is called stochastic. However,the theory of probability can deal only with stochastically stable phenomena. By being stochastically stable, the problem should necessarily have a limit on the sample mean μ_n in a long series of trials. The sample mean is given by the formula: $\mu_n = \frac{1}{n}\sum_{i=1}^{n} x_i$, Where $x_i = 1$ if the phenomenon occurs in the trial and $x_i = 0$ otherwise.

One problem associated with the probability theory in solving uncertainty problems is that it has limited places you can apply it. For instance, one cannot apply it to many economic, environmental, or social problems because we need to perform a large number of trials to ascertain the existence of the limit.

Fuzzy set

Molodtsov (1999) stated that the most appropriate theory for dealing with uncertainties before the soft theory is the theory of fuzzy sets which was developed by Zadeh (1965).

For every set with $A \subset X$, He defined its indicator function $\mu_n as$

$$\mu_n(x) = \begin{cases} 1 \ if x \epsilon A \\ 0 \ if x \notin A \end{cases}$$

The theory is used commonly in different areas such as engineering, medicine and economics, among others. It is based on the fuzzy membership function $\mu: X \rightarrow [0, 1]$. By the fuzzy membership function, we can determine the membership grade of an element to a set. A fuzzy set F is described by its membership function μ_F. Fuzzy set has been used to solve problems in different areas. The difficulty with its use lies in how to set the membership function in each particular case.

Interval mathematics

This is another mathematical tool for dealing with problems of uncertainties. Popper (2002) in attempt to clarify scientific knowledge stated that scientific knowledge is not perfect exactitudes rather it is learning with uncertainty, not eliminating it. He stated that interval mathematics because of the great degree of reliability it provides is usually a part of all other methods that deal with uncertainty. Whenever uncertainty exists, there is always a need for interval computations. Intervals arise naturally in a wide variety of disciplines that deal with uncertain data, including physical measurements, expert estimations, numerical approximations, tolerance problems, economical predictions, quality control techniques, sensitivity analysis, robustness measures of robotic systems, and many others.

The difficulty with this approach however is that they are not sufficiently adaptable for problems with different uncertainties, they cannot be used appropriately to describe smooth changing information, unreliable information, inadequate information and defective information, information with partially contradicting aims and so on (Molodtsov, 1999).

He further presents a Soft set as a collection of approximate descriptions of an object where each approximate description has two parts: a predicate part and an approximate value set. Because The usual mathematical model in classical mathematics is too complicated and we cannot find the exact solution, the notion of approximate solution is introduced here and the solution calculated.

In soft Set Theory, the initial description of the object has an approximate nature, and we do not need to introduce the notion of an exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. We can use any parameterization we prefer with the help of words and sentences, real numbers, functions, mappings, and so on. It means that the problem of setting

the membership function as in fuzzy set or any similar problem does not arise in Soft set Theory.

Basics of Soft set Theory

Molodtsov (1999) in his pioneer work defined soft set:

Definition 1.1 Soft set

Let U be an initial universe set and E a set of parameters to U. Let P (U) denote the power set of U. Suppose also that A is such that A$\subseteq E$.

A pair (F, A) is called a soft set over U if and only if F is a mapping of A into the set of subsets of the set U or the power set of U.

Mathematically: A pair (F, A) is called a soft set over U iff F: A→ $P(U)$,Hence we can write (F, A) as:

$(F, A) = \{F(e) \in P(U): e \in E, F(e) = \emptyset if e \notin A)\}$.

For e ∈ A, F(E) may be considered as the set of $\boldsymbol{e}$ -approximate elements of the soft set (F, A).

A soft set is not a classical Set since a classicalset is a distinct and definite collection, whereas a soft set is the approximate descriptionA soft set over U can be written as (F, A), F_A or (f_A, E).

Definition 1.2

Soft subsets (Maji *et. al*., (2003))

For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) written as (F, A) $\tilde{\subset}$(G, B) if

(i) A ⊂B, and

(ii) $\forall\epsilon \in$ A, F(ϵ) ⊆G (ϵ).

Remark: If (F, A) and (G, B) are soft sets over a common universe U, $(F, A) \tilde{\subseteq} (G, B)$ does not imply that every element of (F, A) is an element of (G, B)(Onyeozili andGwary (2014)).

For example, let U = $\{u_1, u_2, u_3, u_4, u_5\}$ be a universe and E = $\{e_1, e_2, e_3\}$ be a set of parameters such that A = $\{e_1\}$, B = { e_1, e_3 } with (F, A) = { (e_1, $\{u_2, u_4\}$)}, (G, B) = {(e_1, $\{u_2, u_3, u_4\}$), (e_3 , { u_1, u_5})}. It is clear that, ∀ e ∈A; F(e) ⊂ G(e) and A ⊂ B i.e., (F, A) $\tilde{\subseteq}$(G,B). Observe that (e_1,F(e_1)) ∈ (F, A) but (e_1, F(e_1)) ∉ (G, B).

Below is an example of a soft set:

Suppose (F, E) describes the qualities of the lady which Mr. Gido proposes to marry:

Suppose U – is the set of four (4) ladies under consideration

U= $\{l_1, l_2, l_{3,}, l_4\}$

E − is the set of the parameters. And each parameter is a description or quality of the lady.

E = {fair, university graduate, tall, good cook, not divorcee, outspoken, well mannered, Christian, Tiv lady, less than 25}

Represent the parameters with the symbols:

E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$

Where the parameters: e_1=fair, e_2=university graduate, e_3=tall, e_4= good cook, e_5= not divorcee, e_6=outspoken, e_7= good mannered, e_8= Christian, e_9=Tiv(tribe), e_{10}=less than 25}

Let F(e_1)= $\{l_1, l_4\}$

F(e_2) = $\{l_1, l_{3,}\}$

F(e_3) = $\{l_4\}$

F(e_4) = { $l_2, l_{3,}, l_4$}

F(e_5) = $\{l_1, l_2\}$

F(e_6) = {}

F(e_7) = $\{l_1, l_2, l_3\}$

F(e_8) = $\{l_1, l_2, l_{3,}, l_4\}$

F(e_9) = { l_3}

F(e_{10}) = { l_2, l_4}

Hence the soft set (F, E) is a parameterized family $\{F(e_i): i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ of subsets of U.
The soft set (F, E) is as follows:
(F, E) = {fair ladies =$\{l_1, l_4\}$, university graduates ladies = $\{l_1, l_{3,}\}$,
tall ladies = $\{l_4\}$, Good cook ladies = $\{l_2, l_{3,}, l_4\}$, ladies not divorcee = $\{l_1, l_2\}$, outspoken ladies = $\{\}$, good mannered ladies= $\{l_1, l_2, l_3\}$, Christian ladies= $\{l_1, l_2, l_{3,}, l_4\}$, Tiv ladies = $\{l_3\}$, ladies less than 25 years= $\{l_2, l_4\}$}
Onyeozili and Gwary (2014) asserted that the soft set (F, E) can be represented as a set of ordered pairs thus:
(F, A) = $\{(e_1, F(e_1)), (e_2, F(e_2)), (e_3, F(e_3)), (e_4, F(e_4)), (e_5, F(e_5)), (e_6, F(e_6)), (e_7, F(e_7)), (e_8, F(e_8)), (e_9, F(e_9)), (e_{10}, F(e_{10}))\}$ which can further be simplified:
(F, A) = $\{(e_1, \{l_1, l_4\}), (e_2, \{l_1, l_{3,}\}), (e_3, \{l_4\}), (e_4, \{l_2, l_{3,}, l_4\}), (e_5, \{l_1, l_2\}), (e_6, \{\}), (e_7, \{l_1, l_2, l_3\}), (e_8, \{l_1, l_2, l_{3,}, l_4\}), (e_9, \{l_3\}), (e_{10}, \{l_2, l_4\})\}$
In this example to define a soft set means to point out, fair ladies, ladies that are university graduates, tall ladies, etc.
In order to store a soft set in a computer, a two-dimensional table is used to
represent it. Table 1 below shows the tabular representation of the soft set (F, A), such that if $l_i \in F(e_j)$, then l_{ij}= 1, otherwise l_{ij}= 0, where l_{ij} are the entries inthe table below.

Table 1. Tabular representation of a soft set.

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
l_1	1	1	0	0	1	0	1	1	0	0
l_2	0	0	0	1	1	0	1	1	0	1
l_3	0	1	0	1	0	0	1	1	1	0
l_4	1	0	1	1	0	0	0	1	0	1

Applications of Soft set to other Theories
Here we present the applications of Soft set to some other theories.

Applications to Propositional Logic
A compound proposition that is always true regardless of the truth value the individual statements contain is called a tautology. In propositional logic, statements like p→p, p∨¬p, and ¬(p∧¬p) are called tautologies and can be verified by constructing truth tables. On the truth table, they will always contain the true value in the last column. And the converse is true for Contradictions.
Regarding the table 1 as the truth table of the soft set we regard as soft tautologies and soft contradictions the statements that are always true and false respectively as depicted in the table:
$F(e_8) = \{l_1, l_2, l_{3,}, l_4\}$ from the table above is always true hence a soft Tautology. On the contrary, $F(e_6)$ is called a soft contradiction as it is always false.

Applications to the Theory of Fuzzy Set
Let A be a fuzzy set on the universe U, characterized by its membership function μ_A, such thatμ_A: U→ [0, 1],
The family of$\alpha - level$ sets for function μ_A where $\alpha \in [0, 1]$ is given by
$F(\alpha) = \{ x \in U | \ \mu_A(x) \geq \alpha\}$ and $\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$

Thus, the fuzzy set A can be completely defined as a set of ordered pairs given by A = {(x, $\mu_A(x)$): $x \in U$ and $\mu_A(x) \in [0, 1]$
Thus every Zadeh's fuzzy set A, may be considered as the soft set (F, [0, 1]).

Applications to the Theory of Topological Spaces
Given a topological space (X,τ), where τdefines a family of subsets of X, called the open set of X. If F(x) is the family of all open neighborhoods of a point x∈X, i.e, $F(x) = \{v \in \tau | x \in V\}$ then the ordered pair(F, X) is indeed a soft set over X, where F: x→ $P(X)$.
Let X = $\{h_1, h_2, h_3\}$, E = $\{e_1, e_2\}$ and $\tau = \{\emptyset, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), \ldots (F_7, E)\}$
Where.
$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_1, h_2\}$,
$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_1, h_3\}$,
$F_3(e_1) = \{h_2, h_3\}$, $F_3(e_2) = \{h_1\}$,
$F_4(e_1) = \{h_2\}$, $F_4(e_2) = \{h_1\}$,
$F_5(e_1) = \{h_1, h_2\}$, $F_5(e_2)$ = X
$F_6(e_1)$ = X, $F_6(e_2) = \{h_1, h_2\}$,
$F_7(e_1) = \{h_1, h_3\}$, $F_7(e_2) = \{h_1, h_3\}$,
Then (X, τ, E) is a soft topological space over X.

Applications to the Theory of Rough Set (Feng *et. al.*, 2010)
A set X ⊆ U is said to be a Rough set toan equivalence relation R on U , if the boundary region $B_R(X)$ = R*(X) – $R_*(X)$ is non-empty.

Now if we let E and U be the set of parameters and the universe set respectively. And let R be an equivalence relation on U, if F_E is a soft set then we define two soft sets $\vec{F}$: E → $P(U)$ and $\overleftarrow{F}$: E → $P(U)$ as follows:

$\vec{F}(e) = \dot{R}$(F(e)).

$\overleftarrow{F}$(e) = R*(F(e)) for every e ∈ E

We call the soft sets $\vec{F}_E$ and $\overleftarrow{F}_E$, the upper soft set and the lower soft set respectively.

We call the soft set F_E, a Rough soft set if the difference $\vec{F}_E \setminus \overleftarrow{F}_E$ is a non-null soft set and a Crisp soft set if $\vec{F}_E \cong \overleftarrow{F}_E$.

Consider the following example, where a soft set (F, E) describes the conditions of patients suspected of having influenza. The following seven influenza symptoms: respiratory issues, headache, fever, cough, nasal discharge, sore throat, and lethargy. In this example, there were six patients under medical examination in the medical center

U = $\{s_1, s_2, s_3, s_4, s_5, s_6\}$ and E is a set of parameters such as

E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$;

where e_1 = fever, e_2 = respiratory issues, e_3 = nasal discharge, $e_4 =$ cough, e_5 = headache, e_6 = sore throat, and e_7 = lethargy.

Consider the mapping F : E → P(U) given by the following

F(e_1) = $\{s_1, s_3, s_4, s_5; s_6\}$;

F(e_2) = $\{s_1, s_2\}$

F(e_3) = $\{s_1, s_2, s_4\}$

F(e_4) = $\{s_1\}$

F(e_5) = $\{s_1, s_4\}$

F(e_6) = $\{s_1, s_4\}$

F(e_7) = $\{s_1, s_3, s_5, s_6\}$

Therefore, F(e_1) indicates that patients suffer from fever, whose functional value is the set $\{p_1, p_3, p_4, p_5, p_6\}$ Thus, we can view the soft set (F, E) as a collection of approximations. If X = $\{s_1, s_5\}$, then $\underline{SR_P}$X = $\{s_1\}$ and $\overline{SR}_p$X = $\{s_1, s_5, s_6\}$. Hence, $\underline{SR_P}$X ≠ X and X ≠ $\overline{SR}_p$X

Conclusion

Molodtsov (1999), pioneered the idea of a soft set. In this research, we offered various mathematical strategies for dealing with uncertainties and their shortcomings. The concept of a Soft set that is devoid of such deficiencies was introduced, along with some outcomes. The soft set's applications in other theories, such as propositional logic, fuzzy sets, topological spaces, and rough sets, were also addressed.

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